Price Dispersion, Search and the Real Effects of Monetary Policy

Preliminary, Comments Welcome

Nikolay Hristov

University of Augsburg

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1 Introduction

As is well known, the transactions technology in shopping-time models makes money non-neutral even in the case of purely flexible nominal prices. In most cases, however, shopping is modeled and interpreted in a very abstract and *ad hoc* manner. The current paper makes things more explicit by introducing a reason for engaging in search activity in the goods market, which also provide a more concrete interpretation of the resources spent for shopping. In particular, the economy developed here is characterized by price dispersion introduced exogenously *via* differences in productivity across firms. The price dispersion, in turn, generates an incentive for households to make search efforts. The latter lead to transaction costs which can be only financed by real balances accumulated in the previous period. At the same time, a more intense search increases the probability for becoming a customer of a supplier charging relatively low prices. The current paper examines the ability of this modified transactions channel to transmit nominal disturbances to the real economy as well as to make the reactions to monetary surprises (more) consistent with the empirical evidence.

The theoretical framework presented below provides a further rationale for the positive dependence of goods-market search on the current level of inflation found in other models. For example Benabou (1993a,b) develops a menu-cost model with (*S*-*s*)-pricing in which higher inflation increases price dispersion and so makes search in the goods market more profitable. In the inflation-aversion model proposed by Hristov (2008a,b) the positive link between current inflation and search efforts stems directly from the assumed cognitive effect by which search dampens the disutility caused by inflation. In the model developed here a higher inflation erodes the value of individual nominal balances. Therefore, it becomes more important for consumers to find suppliers charging lower prices in order to at least partly compensate the negative effects of the higher inflation rate.

The paper is organized as follows: Section 2 presents the most relevant aspects of the empirical evidence on the reactions to monetary policy shocks. Sections 3 and 4 describe the baseline model and discuss some technical details. The calibration of the model is described in section 5 while section 6 presents the results of the numerical simulations. Section 7 introduces capital accumulation into the baseline model while section 8 is devoted to a comparison between the framework developed in this paper and a (more) standard version of the shopping-time model. Section 9 concludes.

2 Empirical Evidence

Observable Reactions to Monetary Shocks: The VAR-evidence provided by Sims (1980, 1986), Gertler and Gilchrist (1994), Cochrane (1994), Christano *et al.* (1999, 2005), Altig *et al.* (2005), Biovin and Giannoni (2008) and many others indicates that there are delayed, hump-shaped dynamic responses of output, consumption and investment to monetary disturbances, characterized by a substantial degree of persistence with all three variables remaining above their respective initial values for about twelve quarters. To at least partly reproduce the shape and persistence of these impulse responses is of major concern to modern monetary economics.

The Importance of the Cyclical Behavior of Markups: In a comprehensive survey of the empirical studies on the cyclical behavior of prices and marginal costs Rotemberg and Woodford (1999) emphasized the great importance of markups for output fluctuations at business cycle frequencies. According to their results, the output fluctuations attributable to variations of markups, which are orthogonal to fluctuations induced by shifts in the marginal cost curve, account for about 90% of the variance of output growth in the short run.[1] In addition, it can be easily shown that endogenous markup variation on the aggregate level has the potential to substantially magnify (or dampen) business cycles or make them more (or less) persistent.[2] Rotemberg and Woodford (1999) base their estimation on functions of labor share, controlling for variable utilization rates, overhead labor and wages for overtime hours. The authors conclude that markups are negatively correlated with output[3] and respond negatively to demand as well as supply side shocks. Boldrin and Horvath (1996), Gomme and Greenwood (1995), Ambler and Cardia (1996) and Gali *et al.* (2002) also obtain negative estimates of the correlation between output and markups. Also related to the short run fluctuations of marginal costs and markups is the VAR evidence provided by Christiano *et al.* (1999, 2005) and others. They show that an expansionary monetary shock induces an increase in employment[4] and real wages. But if capital is fixed in the short run and there is diminishing marginal product of labor the positive response of employment can be associated with higher real wages only if markups fall. Hence, the impulse responses estimated by Christiano *et al.* can be seen as evidence supporting the findings of Rotemberg and Woodford (1999) about the importance of markup variations as well as their cyclical properties.

[1] Rotemberg and Woodford (1999) decompose output into two components. The fluctuations of the first result solely from shifts in the marginal cost curve for a constant markup while the second component responds only to deviations of markups from their steady state values, and hence represents movements along the marginal cost curve. Rotemberg and Woodford (1999) use the *predicted declines of output* as measure of the cyclical component of output and compare it with the two components of output growth they identify.

[2] Rotemberg and Woodford (1999) provide a simple example.

[3] According to their estimates the correlation between output and markups lies between -0.188 and -0.273.

[4] In fact, Christiano *et al.* (1999, 2005) estimate the impulse responses of output to monetary shocks. But, as capital is a predetermined state variable, increases in output can occur only if hours increase.

The impulse responses implied by the model developed here have the sign predicted by the bulk of the SVAR literature. Unfortunately, their persistence is not consistent with the empirical evidence.

3 The Model

3.1 Theoretical Framework

3.1.1 Goods Market Structure

There are n firms, all producing the same homogeneous good. In each period each household is randomly assigned to one of the suppliers. However, by engaging in search activity in the current period the typical household indexed by i can influence the probability $\tilde{x}_{i,l,t}$, $l = \{1, 2, ..., n\}$ of becoming a customer of each of the individual firms. As a consequence of a higher search intensity, the probability to be assigned to a store with above(below) average price gets smaller(larger) than $1/n$. More formally, $\tilde{x}_{i,l,t}$ is defined as:

$$\tilde{x}_{i,l,t} = \frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_{i,t}^{\gamma}\right)}{\sum_{j=1}^{n} \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_{i,t}^{\gamma}\right)}, \qquad \gamma > 0 \tag{3.1.1}$$

where P_t denotes the overall price level $s_{i,t}$ represents the individual level of search in the goods market. According to this definition, a higher search activity induces an increase (fall) in the probability for becoming a customer of a firm that charges an above average (a below average) current price. Furthermore, $\tilde{x}_{i,l,t}$ is bounded between 0 and 1. Note that the potential or actual price dispersion in this framework can be also seen as an approximation of differences in quality between *almost* homogeneous products selling at the same nominal price. There are two important ideas underlying the definition of $\tilde{x}_{i,l,t}$. First, in spite of the fact that the household is informed about the average price P_t as well as the distribution of individual prices, she doesn't know which supplier offers her the lowest price or the best conditions. In many cases it is not immediately obvious whether two suppliers charging the same price offer the same quality. For example many services such es consulting, banking as well as educational services contain components which are not directly observable. That makes comparisons between individual products costly, as they usually involve the time and resource consuming process of analyzing, tasting, testing and trying different products. Often it is simply not an easy task to find out where the cheapest supplier is located. The service sector again, provides a vast number of examples. Second, I assume that at the end of each period firms randomly change their respective positions within the cross-sectional productivity and thus, the cross-sectional price distribution. As a results, agents are not able to infer from past information, especially from observed past pricing behavior, which firms charge low enough prices and which do not. In other words, firm's movements along the price scala make any knowledge about the past pricing behavior of particular firms worthless, so that at the beginning of an arbitrary period t households are as well informed as they were at the beginning of $t-1$ and thus, have to play the same game again. The assumption on the intra-distribution mobility of firms is based on the evidence provided by Lach (2002) and Lach and Tsiddon (1993). After controlling for observed as well as unobserved heterogeneity between almost identical products they show that there is substantial intra-distribution mobility disabling, as the authors conclude, consumers to learn which store charges consistently low prices

As explained below, under the assumption of a continuum of *ex ante* symmetric households which are able to perfectly pool all idiosyncratic income as well as expenditure risks the mass of households served by an arbitrary firm l, $x_{l,t}$ will be equal to the probability to become a customer at store l faced by a typical household:[5]

$$x_{l,t} = \tilde{x}_{i,l,t}.$$

I refer to $x_{l,t}$ as the market share of firm l. Since, as assumed below, it is costly to search for cheaper suppliers, deviations from the average price do not translate into an immediate drop or increase of the individual market share to zero or hundred percent respectively. Thus, each firm enjoys a small, short-run monopoly power over the consumers belonging to its customer base when setting its price. Consequently, the market structure can be characterized as a form of monopolistic competition. According to (3.1.1), if all firms were to choose the same price the fraction of aggregate demand each firm faces would be equal to $1/n$, irrespective of the level of search activity. If households do not engage in search at all, $s_{i,t} = 0$, $\forall i$ then again each supplier will serve a fraction of $1/n$ of the market, irrespective of the degree of price dispersion.

In contrast to the versions of the *Customer Market Model* described in Hristov (2008a, 2008b, 2008c) where the individual firm's market share follows a random walk process, in the model presented here the market share $x_{l,t}$ is modeled as a variable *without memory*, which is purely statically related to the level of search efforts and the firm's relative prices. The assumption that the process of search, the price adjustments and the reactions of the individual market shares take place simultaneously can be regarded as reasonable, since one period in the model corresponds to one quarter in the real world. ?????

3.1.2 Firms

Each profit maximizing monopolistic firm produces according to the linear production function

$$Y_{l,t} = (Z_t + \iota_{l,t})N_{l,t},$$

[5] That an implication of the *Law of Large Numbers*.

where $N_{l,t}$ denotes labor input of firm l. Z_t denotes the total factor productivity which follows a stochastic process given by:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t,$$

where ϵ_t follows a *White Noise Process* with variance σ_ϵ^2. $\iota_{l,t}$ is an exogenous firm specific shift variable which is assumed to evolve according to

$$\iota_{l,t} = \iota_l + \epsilon_{\iota_l,t}.$$

$\epsilon_{\iota_l,t}$ follows a *White Noise Process* with variance $\sigma_{\iota_l}^2$.

The demand function faced by the producer l is given by

$$D_{l,t} = x_{l,t} \cdot \frac{D_t}{n} = \left(\frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^\gamma\right)}{\sum_{j=1}^n \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^\gamma\right)} \right) \cdot \frac{D_t}{n}, \tag{3.1.2}$$

where D_t, s_t and P_t denote, respectively, aggregate demand, aggregate search efforts and the aggregate price level. For given marginal costs, $\mu_{l,t}$ the profit maximization problem of a typical firm reads:

$$\max_{\frac{P_{l,t}}{P_t}} \left\{ \left(\frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^\gamma\right)}{\sum_{j=1}^n \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^\gamma\right)} \right) \cdot \frac{D_t}{n} \left(\frac{P_{l,t}}{P_t} - \mu_{l,t} \right) \right\}.$$

It yields the following first order condition for optimal price setting:

$$\frac{P_{l,t}}{P_t} = \frac{1}{s_t^\gamma (1 - x_{l,t})} + \mu_{l,t}, \tag{3.1.3}$$

where $x_{l,t}$ also depends on $\frac{P_{l,t}}{P_t}$. Everything else given, a higher level of search efforts makes the typical firm more reluctant to set too high a relative price, and leads to a lower markup. A higher market share $x_{l,t}$ makes it less likely for a searching customer to meet a supplier other than l and thus, makes it less likely for him to find a firm charging a price lower than $P_{l,t}$. Consequently, a higher $x_{l,t}$ reduces the magnitude of the negative effect of any given level of search on the relative price $\frac{P_{l,t}}{P_t}$ and so enables firm l to choose a higher markup.

However, by inspecting equation (3.1.3) one can only gain some very rough intuition about the mechanisms underlying the price setting behavior of the firms, because (3.1.3) defines an implicit relationship between the relative price $\frac{P_{l,t}}{P_t}$, marginal costs $\mu_{l,t}$ and search efforts s_t.

Log-linearizing (3.1.3) around the steady state[6] yields:[7]

$$\left(\widehat{\frac{P_{l,t}}{P_t}}\right) = \frac{\gamma}{s^{\gamma}\cdot\frac{P_l}{P}}\left(\frac{\left(1-\frac{P_l}{P}\right)s^{\gamma}-(1-x_l)}{1-x_l}\right)\hat{s}_t + \\ + \frac{x_l}{(1-x_l)\cdot\frac{P_l}{P}}\cdot\sum_{j=1,j\neq l}^{n} x_j\frac{P_j}{P}\left(\widehat{\frac{P_{j,t}}{P_t}}\right) + (1-x_l)\frac{\mu_l}{\frac{P_l}{P}}\hat{\mu}_{l,t}, \tag{3.1.4}$$

where variables without time index denote steady state values, while a "hat", " ^ ", over a variable denotes its percentage deviation from the stationary equilibrium. According to (3.1.4), if firm l has an above average steady state price, an increase of search activity will force it pass-through to its price a smaller fraction of any given increase in its marginal costs and thus, to lower its markup. Only in the case of a sufficiently low P_l/P combined with a sufficiently large market share, x_l will an increase in aggregate search activity enable firm l to choose a higher pass-through and increase its markup. Putting any general equilibrium effects aside, if all other firms increase their respective relative prices then firm l will also find it optimal to do that. Note that in the general equilibrium discussed in the current paper it will be possible for all firms to simultaneously increase their respective relative prices, provided that such a reaction comes along with (is backed by) the *"correct"* adjustment of the individual market shares.

The more conventional representation of the first order condition for optimal price setting is:

$$\frac{P_{l,t}}{P_t} = mu_{l,t}\mu_{l,t} = \left(1+\frac{1}{s_t^{\gamma}(1-x_{l,t})\mu_{l,t}}\right)\mu_{l,t},$$

where $mu_{l,t}$ denotes the firm specific markup. As can be easily seen, it will be time varying. As stressed above, the markup will be only procyclical when P_l/P is sufficiently low and at the same time x_l sufficiently large. Otherwise, $mu_{l,t}$ will tend to be countercyclical.

Since labor is the only factor of production, the real marginal costs of firm l are given by

$$\mu_{l,t} = \frac{W_t/P_t}{Z_t+\iota_{l,t}}. \tag{3.1.5}$$

[6] The properties and the computation of the steady state will be discussed later on.

[7] To arrive at the result one has just to take into account the definition of $x_{l,t}$, $\forall l$, the fact that

$$\sum_{j=1}^{n} x_{j,t} = 1$$

and the definition of the price index:

$$P_t = \sum_{j=1}^{n} x_{j,t}P_{j,t},$$

and then to log-linearize.

3.1.3 Households

The economy is populated by a continuum of *ex ante* identical agents of total mass equal to one, organized in m equally large units. Let us refer to these units as *families*. The family indexed by $j = \{1, 2, ..., m\}$ consists of the agents with an index i in the interval $i \in \left[\frac{j-1}{m}, \frac{j}{m}\right]$. Let us call them *families*. Each family faces a two-stage maximization problem, in which both stages take place simultaneously. At the *"first"* stage, the head of the family, also called planner, chooses the level as well as the distribution across the family's members of next-period wealth, current income, leisure and current expenditure, given the level and the distribution of the family's wealth, average prices and the current level of search activity. The j^{th} planner maximizes the following utility function:

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{C_{i,t}^{1-\eta}}{1-\eta} + \phi \frac{(M_{i,t}/P_t)^{1-\chi}}{1-\chi} - \frac{b}{2} N_{i,t}^2 \right) \right\}, \qquad \phi, b, \eta, \chi > 0, \quad \beta \in (0,1),$$

subject to the budget constraint:

$$\int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\sum_{l=1}^{n} \tilde{x}_{i,l,t} \frac{P_{l,t}}{P_t} D_{i,t} + \frac{M_{i,t+1}}{P_t} - \frac{M_{i,t}}{\pi_t P_{t-1}} \right) = \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{W_t}{P_t} N_{i,t} + \Pi_t + \frac{T_t}{P_t} \right), \quad \forall t,$$

where $D_{i,t}$, $M_{i,t}$ and $N_{i,t}$ denote the agent specific total expenditure, nominal balances and working hours respectively. W_t, Π_t and T_t denote the nominal wage, real profits and nominal net transfers form the government respectively. π_t represents the overall inflation factor. The total expenditure of the i^{th} member of the family, $D_{i,t}$, equals the sum of her consumption expenditure $C_{i,t}$ and the transaction costs $g(s_{i,t}) \frac{M_{i,t}}{\pi_t P_{t-1}}$ arising when setting search efforts at $s_{i,t}$.

At the *"second"* stage, given average prices as well as the distribution of the family's wealth, income, leisure and consumption expenditure, each member of the family chooses the level of its search efforts in order to buy the amount of goods chosen by the planner at the lowest possible cost. The corresponding maximization problem can be written as:

$$\max_{s_{i,t}} \left\{ -\sum_{l=1}^{n} \tilde{x}_{i,l,t}(s_{i,t}) D_{i,t}(s_{i,t}) \right\}, \tag{3.1.6}$$

where $\tilde{x}_{i,l,t}$ is defined in (3.1.1).

The two-stage structure can be seen as an approximation of the process of decision-making in many families, corporations, public and private institutions and other economic units and is consistent with the approach chosen by a large part of the *New Home Economics* literature. The latter views the family as a social unit in which one of the members, usually the husband, acts as a benevolent planner, endowed with dictatorial power over the other members of the family, who pools the income streams of the individual members and seeks to maximize a kind of *"social welfare function"* defined as a weighted average of the utility functions of the family's members. Each of them then chooses a set of variables (e.g. the production of particular home goods) to maximize her individual objective function. Examples are Becker (1973, 1974), Killingsworth (1983), Lundberg and Pollak (1997) as well as the literature cited there. In most of those models the decision to become (remain) a member of the family is endogenous and shapes the behavior of the *"dictator"*. In the current paper it is assumed that for reasons exogenous to the model neither agent has an incentive to leave the family she belongs to.

Many corporations and other public and private institutions are similarly organized: There is a lot of dictatorship and centralized planning taking place in them, with the managers of such institutions making almost all important decisions and delegating only the ones of limited importance to the individual departments of their institution. In a panel study including 300 large U.S. firms Rajan and Wulf (2006) do find support of the widespread view that the organizational hierarchy of the U.S. firms has become flatter over the last twenty years. The process has been characterized by the elimination of many intermediate layers of management and a declining organizational distance between the CEO and the division managers. However, the authors point out that a naive interpretation of the observed organizational flattering as a "decentralization" might be incorrect because on the one hand, decision-making authority is being delegated down to the individual division heads but on the other hand, the CEO is getting a more direct control over the lower levels of the organization, which is a form of centralization. Rajan and Wulf (2006) further conclude that despite the organizational flattering found in the data, it is still the case that the CEO and the members of the senior management are the ones *'...who make the resource allocation decisions that ultimately determine the firm's performance...'*. In an excellent essay Argyris (1998) argues that in spite of the observable organizational flattering, most companies are still dictatorially governed without much *empowerment* of division managers and other employees actually being done. The reasons are, as both, research and practice, indicate, that on the one hand the *'command-and-control model'*[8] is what CEOs know best and on the other, most employees find it disadvantageous when being held personally accountable. Not surprisingly, the author comes to the conclusion that decentralization and empowerment in firms are just *'the emperor's new clothes'*.[9] Further examples for theoretical studies discussing recent developments in the organizational structure of firms and deriving similar conclusions are Kaplan (1996), Holmstrom and Kaplan (2001), Rajan and Zingales (2000) and many others.

Transaction costs in this model constitute a fraction of nominal money balances accumulated in the previous period. By this specification I attempt to capture the fact that for performing

[8] See Argyris (1998), p. 98.

[9] See Argyris (1998), p. 104.

different tasks the departments of many public and private institutions as well as the members of many families are funded in nominal terms *in advance*. Given that more or less fixed nominal (monthly, quarterly or annual) budget the departments and family members carry out different types of transactions. An alternative rationale for the specification chosen, can be found by assuming that there are autonomous business units (or firms) performing search in the goods market and selling the information obtained for money to the households.[10] Loosely speaking, both interpretations can be viewed as a kind of a *cash in advance* constraint with respect to search activity.

Assume for now that each member chooses the same level of search activity, $s_{i,t} = s_t$, $\forall i$. It is then easy to show that the optimal allocation from the point of view of the planner will be symmetric in any respect across the family members. Given that symmetry and applying the law of large numbers implies the following relation between the market shares of the individual firms $x_{l,t}$ and the agent specific probabilities to become a customer of firm l, $\tilde{x}_{i,l,t}$:

$$x_{l,t} = \tilde{x}_{i,l,t}, \quad \forall i, l.$$

Then by using the definition of the overall price index:

$$\sum_{j=1}^{n} x_{j,t} \frac{P_{j,t}}{P_t} = 1$$

the budget constraint of family j can be written as

$$\int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(C_{i,t} + \frac{M_{i,t+1}}{P_t} - (1 - g(s_{i,t})) \frac{M_{i,t}}{\pi_t P_{t-1}} \right) = \int_{i=\frac{j-1}{m}}^{\frac{j}{m}} \left(\frac{W_t}{P_t} N_{i,t} + \Pi_t + \frac{T_t}{P_t} \right), \quad \forall t.$$

The last equation shows that, as a consequence of the symmetry assumption, the typical family does not face any idiosyncratic risks although each individual member is exposed to individual uncertainty with respect to her current expenditure. If in addition initial wealth is distributed uniformly across families, the latter will be homogeneous. As a result, one will be able to resort to the representative agent framework. The latter is certainly less realistic than a similar model with heterogeneous households would be, but since I am only interested in the cyclical behavior of the most important macroeconomic aggregates and not in that of individual variables, the loss of relevant information caused by the symmetry assumption will be negligible. Furthermore, the representative agent approach has the advantage of involving a much lower computational burden than the heterogeneous agent framework does. The empirical literature dealing with the extent to which there is income pooling within families provides mixed evidence. Lundberg and Pollak (1997) review that literature and reject in their own study the hypothesis of income pooling by exploiting a natural experiment found in the data.

[10] Under this specification the overall price index will be a weighted average of the prices of the consumption goods and the price charged for performing search activity.

3.1.4 First Order Conditions

The first order condition resulting from the first stage of optimization performed by the planner, evaluated at the symmetric family specific equilibrium, take the form:

$$C_t^{-\eta} = \Lambda_t, \tag{3.1.7}$$

$$bN_t = \Lambda_t \frac{W_t}{P_t}, \tag{3.1.8}$$

$$\beta\phi m_{t+1}^{-\chi} E_t\{\pi_{t+1}^{\chi-1}\} = \Lambda_t(1-g(s_t)) - \beta E_t\left\{\frac{\Lambda_{t+1}(1-g(s_{t+1}))}{\pi_{t+1}}\right\}, \tag{3.1.9}$$

$$C_t + m_{t+1} - (1-g(s_t))\frac{m_t}{\pi_t} = \frac{W_t}{P_t}N_t + \Pi_t + \frac{T_t}{P_t}. \tag{3.1.10}$$

The family index was dropped from C_t, N_t , $m_t = \frac{M_t}{P_{t-1}}$ and s_t because of the homogeneity across the families in this economy.

The second stage of maximization implies the following first order condition:

$$\frac{g'(s_{i,t})s_{i,t}^{1-\gamma}}{\gamma} = \left(\sum_{l=1}^{n} \tilde{x}_{i,l,t}\left(1-\frac{P_{l,t}}{P_t}\right)^2\right)\frac{\pi_t}{m_{i,t}}D_{i,t},$$

where i denotes the index of the family member. Note that $\tilde{x}_{i,l,t}$ and $D_{i,t}$ also depend on $s_{i,t}$. Evaluating the last equation at the symmetric equilibrium and specifying $g(s_{i,t})$ as

$$g(s_{i,t}) = \frac{s_{i,t}^{\alpha}}{a}, \quad \alpha, a > 0, \quad \alpha > \gamma,$$

yields the condition

$$\frac{\alpha}{a\gamma}s_t^{\alpha-\gamma} = \left(\sum_{l=1}^{n} x_{l,t}\left(1-\frac{P_{l,t}}{P_t}\right)^2\right)\frac{\pi_t}{m_t}D_t. \tag{3.1.11}$$

This equation embodies the optimal trade off between the additional increase in transaction costs and the corresponding additional reduction of the average goods price, both brought about by a marginal increase in search activity s_t. Recall that for given relative prices a higher s_t implies a lower (higher) probability to become a customer of a relatively expensive (cheap) supplier. According to (3.1.11), if there is no price dispersion, search activity will be zero. To avoid the mathematical and computational complications stemming from the possibility of such a corner solution, the exogenous productivity processes Z_t and $\iota_{l,t}$, $l = \{1, 2, ..., n\}$ are calibrated so as to ensure that at each point in time and in each state of nature there is a non-degenerate distribution of goods prices.[11] For given total expenditure D_t and market shares

[11] Note further that if there is no search activity $s_t = 0$, each firm will be able to set its price at infinity since:

$$\frac{P_{l,t}}{P_t} = \frac{1}{s_t^{\gamma}(1-x_{l,t})} + \mu_{l,t}.$$

$x_{l,t}$ a higher inflation π_t reduces the transaction costs *per unit of* s_t, thus, forcing households to increase their search efforts. It is not easy to provide a more general intuition about that dependance, an intuition which is sufficiently close to reality. Perhaps it is plausible to assume that in times the economic conditions are worsening, and an increase inflation does represent a worsening since it erodes the value of individual nominal balances, it becomes more important for consumers to find suppliers charging lower prices in order to at least partly compensate the negative effects of the higher inflation rate. Perhaps it is plausible to assume that in such episodes households get more sensitive to differences in prices and are willing to take a more careful look at the price setting behavior in the goods and other markets. A similar effect arises in the class of monetary models known as *Shopping Time Models*.[12] The latter motivate the demand for real balances by the desire to reduce the transaction costs coming about with the purchase of consumption goods. A higher inflation rate in that models necessitates a higher fraction of time spent in the production of transaction services or the time spent carrying out transactions. Put differently, a higher inflation in the shopping time models reduces the disutility of any given amount of transaction time.

3.1.5 Government

The central bank finances its lump-sum transfers to the public by changes in the nominal quantity of money:

$$M_{t+1} - M_t = T_t.$$

It is further assumed that in each period transfers constitute a fraction of current money supply:

$$T_t = (\tau_t - 1)M_t,$$

Such a reaction would lead to a collapse of the economy. To avoid this, one could replace s_t^γ in the definition of the market share $x_{l,t}$ by the function $1 + s_t^\gamma$. This modification, however, reduces the set of parameter combinations implying an economically meaningful steady state. I performed a large number of numerical experiments in order to compare the qualitative and quantitative implications of both specifications, the one with s_t^γ and the one with $1 + s_t^\gamma$. The result was that, provided the parameterization of the model leads to an economically interpretable steady state, the two specifications imply virtually identical results. Nevertheless, it should be noted that the model developed here is suitable only for the analysis of economies characterized by some price dispersion in each period and each state of nature. To understand the evolution of economies which can switch from an asymmetric to a symmetric equilibrium and *vice versa*, one should resort to another theoretical tools.

[12] Examples are Saving (1971), Brock (1974), Croushore (1993), Jovanovich (1982), Romer (1986) and many others.

where the percentage deviation of τ_t from its steady state $\hat{\tau}_t$ follows a first order autoregressive process

$$\hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + u_t, \qquad \rho_\tau \in [0, 1).$$

u_t is assumed to be a *White Noise Process* with variance σ_u^2.

3.1.6 Equilibrium

The evolution of the economy is described by the definition of marginal costs (3.1.5), the first order condition for optimal price setting (3.1.3), the households first order conditions (3.1.7), (3.1.8), (3.1.9) and (3.1.11), the aggregate consistency conditions

$$x_{l,t} D_t = (Z_t + \iota_{l,t}) N_{l,t}, \qquad l = \{1, 2, ..., n\}, \tag{3.1.12}$$

and

$$\sum_{l=1}^{n} N_{l,t} = N_t \tag{3.1.13}$$

as well as the definitions of $x_{l,t}$ for $l = \{1, 2, ..., n\}$ and D_t. Note that if (8.2.4) are satisfied, then the family's budget constraint implies:[13]

$$D_t = Y_t = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t} (Z_t + \iota_{l,t}) N_{l,t}.$$

To close the model, one also needs to specify monetary policy and the exogenous productivity processes Z_t and $\iota_{l,t}$.

The inclusion of search activity s_t as an argument of the function describing the evolution of firm-specific market share introduces an externality from the point of view of the individual firm, since s_t depends on overall inflation and consumption.

4 Technical Discussion

Without any symmetry assumptions the expenditure minimization carried out in the second stage of utility maximization delivers the following first order condition:

$$\frac{\alpha}{\gamma a} s_{i,t}^{\alpha-\gamma} = \left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{\pi_t}{m_{i,t}} D_{i,t}, \tag{4.0.14}$$

[13]Note that the goods market equilibrium together with the definition of the price index imply:

$$\sum_{l=1}^{n} \frac{P_{l,t}}{P_t} (Z_t + \iota_{l,t}) N_{l,t} = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t} x_{l,t} D_t = D_t.$$

where $\Upsilon_{i,t}$, defined as

$$\Upsilon_{i,t} = \sum_{l=1}^{n} \tilde{x}_{i,l,t} \frac{P_{l,t}}{P_t},$$

is the price index perceived by member i of an arbitrary family. If equation (4.0.14) has a unique solution $s^*_{i,t}$, then it will be straightforward to show that, provided the family planner chooses a symmetric allocation $m_{i,t} = m_t$ and $C_{i,t} = C_t$, $\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$, each family member will find the same level of search efforts optimal: $s_{i,t} = s_t$, $\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$. Otherwise, symmetry of the planner's allocation won't necessary imply symmetry with respect to search activity. Thus, it is important to identify conditions ensuring that equation (4.0.14) has only one solution. Surely, a more challenging goal would be to characterize the whole set of necessary and sufficient conditions for uniqueness. Unfortunately, the latter is not possible due to the high degree of nonlinearity in the model.

(4.0.14) can be more explicitly written as:

$$\left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{\pi_t}{m_{i,t}} C_{i,t} + \left(\frac{\sum_{l=1}^{n} \left\{ \left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \right\}}{\Upsilon_{i,t}} \right) \frac{s^{\alpha}_{i,t}}{a} =$$

$$= \frac{\alpha}{\gamma a} s^{\alpha-\gamma}_{i,t}. \tag{4.0.15}$$

First observe that the first derivative of $\Upsilon_{i,t}$ with respect to the agent's search activity $s_{i,t}$ is negative. Then it is easy to see that an increase of $s_{i,t}$ will have a positive effect on the lhs of equation (4.0.15) *via* the common denominator of the two terms, $\Upsilon_{i,t}$, and $s^{\alpha}_{i,t}$ appearing in the second term. Unfortunately, there is an ambiguous effect of $s_{i,t}$ on the numerators of the expressions in brackets on the lhs of (4.0.15). To see that, note that the product $\left(1 - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t}$ is positive for some l and negative for others. Thus, the sign of $\left(1 - \frac{P_{l,t}}{P_t}\right) \tilde{x}_{i,l,t} \frac{\partial \Upsilon_{i,t}}{\partial s_{i,t}}$ will depend on l. Further, since the sign of $\frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ also depends on the firm's index l, the sign of $\left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right) \frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ will also be ambiguous.[14] Because of the high degree of nonlinearity of equation (4.0.15) it is almost impossible to derive any interpretable, general conditions on the derivatives $\frac{\partial \tilde{x}_{i,l,t}}{\partial s_{i,t}}$ and the degree of price dispersion ensuring a unique solution $s^*_{i,t}$. Therefore I restrict the analytical and numerical analysis to the case $n = 2$.

[14]Note that in most cases the sign of the product $\left(1 - \frac{P_{l,t}}{P_t}\right) \left(\Upsilon_{i,t} - \frac{P_{l,t}}{P_t}\right)$ will be positive since $\Upsilon_{i,t}$ will tend to take a value near one.

If there are only two suppliers in the goods market, $n = 2$, (4.0.15) simplifies to

$$0 = \left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)^2 \frac{e_{i,1,t}e_{i,2,t}}{\left(e_{i,1,t} + e_{i,2,t}\right)\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)} \left(C_{i,t} + \frac{s_{i,t}^{\alpha}}{a}\frac{m_{i,t}}{\pi_t}\right)\frac{\pi_t}{m_{i,t}} - \frac{\alpha}{\gamma a}s_{i,t}^{\alpha-\gamma}, \tag{4.0.16}$$

where

$$e_{i,1,t} = \exp\left(\left(1 - \frac{P_{1,t}}{P_t}\right)s_{i,t}^{\gamma}\right), \qquad e_{i,2,t} = \exp\left(\left(1 - \frac{P_{2,t}}{P_t}\right)s_{i,t}^{\gamma}\right).$$

Without loss of generality it can be assumed that $P_{2,t} > P_{1,t}$ holds. A sufficient condition for (4.0.16) to have at most one solution $s_{i,t}^{*}$ is that the first derivative of its lhs with respect to $s_{i,t}$ is negative for all $s_{i,t} > 0$. This first derivative is given by

$$\left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)^2 \frac{e_{i,1,t}e_{i,2,t}}{\left(e_{i,1,t} + e_{i,2,t}\right)\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)} \times$$

$$\times \left\{\gamma s_{i,t}^{\gamma-1}\frac{\frac{P_{1,t}}{P_t}\left(\frac{P_{1,t}}{P_t} - \frac{P_{2,t}}{P_t}\right)e_{i,1,t}^2 + \frac{P_{2,t}}{P_t}\left(\frac{P_{2,t}}{P_t} - \frac{P_{1,t}}{P_t}\right)e_{i,2,t}^2}{\left(e_{i,1,t} + e_{i,2,t}\right)\left(e_{i,1,t}\frac{P_{1,t}}{P_t} + e_{i,2,t}\frac{P_{2,t}}{P_t}\right)}\left(C_{i,t}\frac{\pi_t}{m_{i,t}} + \frac{s_{i,t}^{\alpha}}{a}\right) + \alpha\frac{s_{i,t}^{\alpha-1}}{a}\right\}, \tag{4.0.17}$$

and its sign depends on the sign of the expression in curly brackets. A necessary condition for it to be negative is that the following inequality is satisfied:

$$\frac{P_{1,t}}{P_{2,t}} > \left(\frac{e_{i,2,t}}{e_{i,1,t}}\right)^2 = \frac{1}{\exp\left(2s_{i,t}^{\gamma}\left(\frac{P_{2,t}}{P_t} - \frac{P_{1,t}}{P_t}\right)\right)}, \quad \forall s_{i,t} > 0. \tag{4.0.18}$$

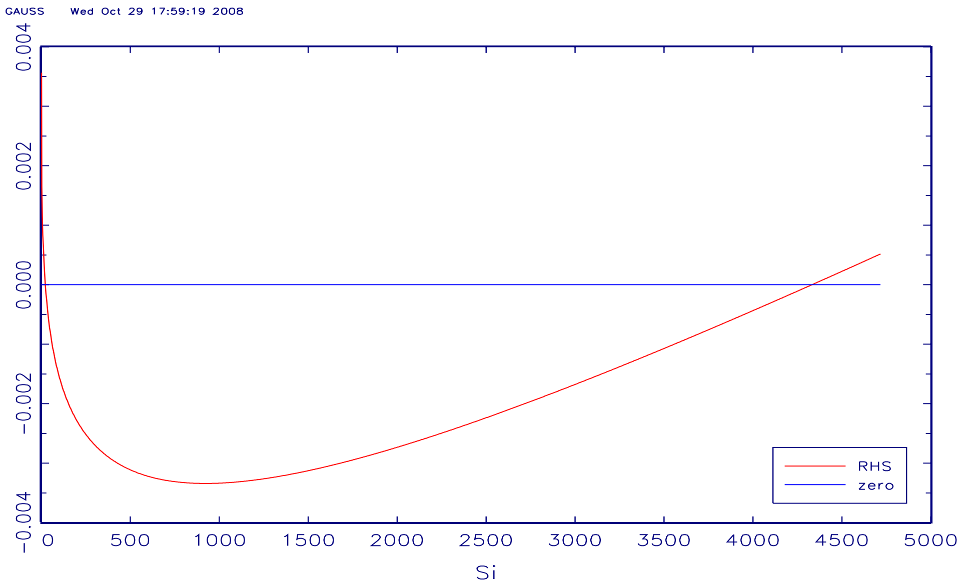
When (4.0.18) holds and at the same time the velocity of money with respect to consumption $C_{i,t}\frac{\pi_t}{m_{i,t}}$ is sufficiently large, then the sum in curly brackets in (4.0.17) will be negative. This requirements can be used to derive a sufficient condition on the parameters of the model provided one is able to find the solutions for the endogenous variables involved. However, as is readily confessed, due to the high degree of nonlinearity in this model it is not possible to derive such a condition analytically. Alternatively, one can resort to numerical analysis and try to derive a restriction on the parameters and the steady state of the model heuristically. Such a restriction will ensure that in the stationary as well as in the relevant temporary equilibria equation (4.0.16) has a unique solution. If (4.0.17) turns to be positive, it will be possible for equation (4.0.16) to have multiple solutions $s_{i,t}^{*}$.

To examine the existence and the uniqueness of the solution $s_{i,t}^{*}$ of (4.0.16) numerically,[15] one needs to calibrate α, γ, a, the velocity of money with respect to output $v_y = Y\frac{\pi}{m}$,

[15] The corresponding program is "**equilibrium_2a.g**".

working hours N and both relative prices, P_1/P and P_2/P. The benchmark values for the velocity v_y, N, P_1/P and P_2/P are set to 2.15, 0.13, 0.97 and 1.08 respectively. Their computation is described in section 5. As also shown in section 5, given the values of α, γ, N, v_y and both relative prices one is in a position to compute the parameter a. In the numerical investigation of the properties of equation (4.0.16) I perform a sensitivity analysis by experimenting with values of α, γ, v_y as well as the difference $P_2/P - P_1/P$ different from their respective benchmark levels. v_y ranges between 0.5 and 4.3, α and γ take values in the intervals $[0.4, 20]$ and $[0.3, 19.8]$ respectively, whereas γ is always smaller than α. The values of the difference between P_2/P and P_1/P cover a bounded open interval the determination of which is described in section (5).

Figure iv.1: Rhs of equation (4.0.16). $\frac{P_1}{P} = 0.97$, $\frac{P_2}{P} = 1.08$, $\frac{Y\pi}{m} = 2.15$, $\alpha = 0.9$, $\gamma = 0.7$.



RHS - rhs of equation (4.0.16), *Si* - level of individual search activity.

Fortunately, the results are readily summarized. It turns out that for all parameter combinations considered equation (4.0.16) has exactly two solutions $s^*_{i,t}$ and $s^{**}_{i,t}$ with $s^*_{i,t} < s^{**}_{i,t}$. Figures iv.1 and iv.2 depict the rhs of this equation for two sets of parameter values. The smaller solution always implies that the fraction of real balances used for transaction purposes $g(s^*_{i,t})$ lies in the interval $(0, 1)$ and is thus, consistent with the structure of the model. The larger one $s^{**}_{i,t}$ however, implies either $g(s^{**}_{i,t}) > 1$ or $g(s^{**}_{i,t}) \in (0, 1)$ depending on the particular parameter values. If the fraction $g(s^{**}_{i,t})$ turns to be larger than one, then we can disregard $s^{**}_{i,t}$ as a solution with no economic interpretation. In this case we are left with only one economically meaningful solution. As a consequence of this uniqueness, if the family planner allocates initial wealth $m_{i,t}$ and consumption expenditure $C_{i,t}$ symmetrically across family members, then each of them will choose the same level of search activity, $s_{i,t} = s_t$,

$\forall i \in [\frac{j-1}{m}, \frac{j}{m}]$. If the following inequalities hold:

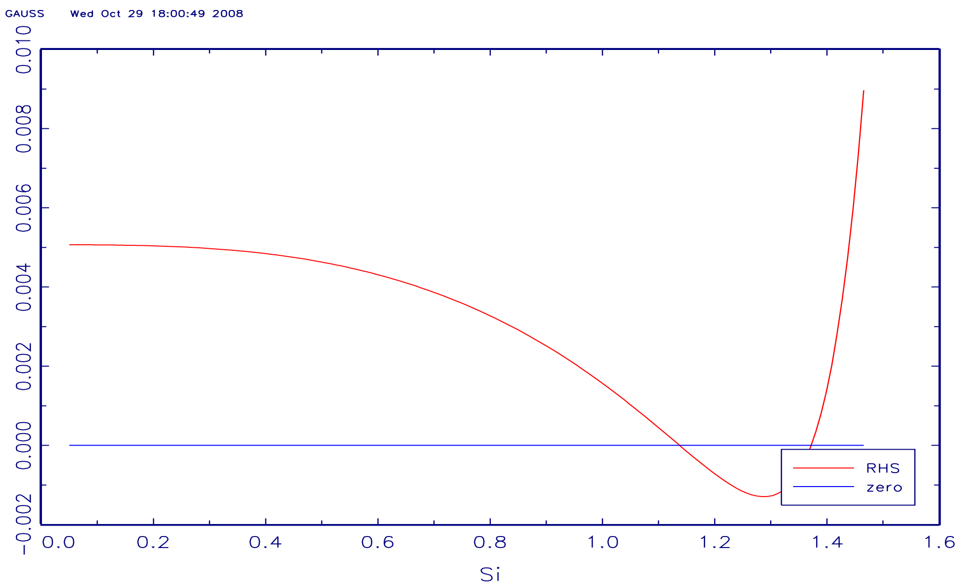
$$g(s^{*}_{i,t}) \in (0,1), \qquad g(s^{**}_{i,t}) \in (0,1),$$

there will be no economic reason for ignoring one of the solutions. This kind of multiplicity implies that in general agents will be heterogeneous with respect to the level of their search efforts even if the planner were to distribute money balances and consumption uniformly across the family members. It turns out that the higher the price dispersion $P_2/P - P_1/P$ and the smaller the difference between α and γ the more likely for multiple solutions to exist. In contrast, the higher the velocity of money v_y the larger the probability for $g(s^{**})$ to be grater than one and thus, the more likely for the economically relevant solution to (4.0.16) to be unique. The cases depicted in figures iv.1 and iv.2 are both characterized by the inequality

$$g(s^{**}_{i,t}) > 1$$

implying that $s^{**}_{i,t}$ can be ignored.

Figure iv.2: Rhs of equation (4.0.16). $\frac{P_1}{P} = 0.97$, $\frac{P_2}{P} = 1.08$, $\frac{Y\pi}{m} = 2.15$, $\alpha = 20$, $\gamma = 17$.



RHS - rhs of equation (4.0.16), *Si* - level of individual search activity.

In the simulations presented below I use only calibrations of the model ensuring that (4.0.16) has only one interpretable solution. Such a calibration is very easy to find since the bulk of the parameter combinations examined in this section lead to $g(s^{**}_{i,t}) > 1$. It is important to note, that the uniqueness of the solution of (4.0.16) does not necessary imply uniqueness of the stationary or any temporary equilibrium of the model. In the current paper I concentrate on the symmetric equilibrium because in my view it is the most likely and most plausible one, provided that the families and their members are *a priori* homogeneous. I do not make an attempt to prove the existence or non-existence of further equilibria, characterized by an asymmetric distribution of resources and heterogeneous levels of search activity, and leave this issue for future research. In this paper I am only able to show numerically that for the calibrations chosen the symmetric equilibrium is *locally unique*.

5 Calibration

The calibration of this model is more involved than it was the case in the models already presented.[16] First, to reduce the computational burden arising in the approximation, the calibration and the simulation steps, I consider only the 2-firms case. I assume that the steady state of the economy is characterized by price dispersion with $\frac{P_1}{P}$ always being smaller than $\frac{P_2}{P}$. Alternatively one can assume that the difference between the two relative prices remains constant over time while in each period firms randomly switch their positions in the price distribution. The definition of the overall price index

$$x_1 \frac{P_1}{P} + x_2 \frac{P_2}{P} = 1, \qquad x_1 \in (0, 1), \quad x_2 = 1 - x_1$$

implies that $\frac{P_1}{P} < 1$ and $\frac{P_2}{P} > 1$ hold. I start the calibration by setting the difference between both relative prices Δ as well as the lower one at particular values. Then the definition of the price index allows us to determine x_1:

$$x_1 \frac{P_1}{P} + (1 - x_1) \left(\frac{P_1}{P} + \Delta \right) = 1,$$

$$\Rightarrow$$

$$1 - x_1 = \frac{1 - P_1/P}{\Delta}.$$

To ensure that the market share of the cheaper supplier is larger then 50 percent, Δ should satisfy the following inequality

$$\Delta > \frac{1 - P_1/P}{0.5}. \tag{5.0.19}$$

Otherwise the firm charging the higher price will enjoy a larger market share. The level of search activity in the stationary equilibrium s is identified as the solution of the following equation:

$$\frac{\exp\left(\left(1 - \frac{P_1}{P} - \Delta\right) s^{\gamma}\right)}{\exp\left(\left(1 - \frac{P_1}{P}\right) s^{\gamma}\right) + \exp\left(\left(1 - \frac{P_1}{P} - \Delta\right) s^{\gamma}\right)} = \frac{1 - P_1/P}{\Delta},$$

[16] The corresponding programs are "**equilibrium_2.g**" for the general analysis of the sensitivity of the steady state with respect to the degree of price dispersion $\Delta = P_2/P - P_1/P$ and "**equilibrium_2a.g**" for the determination of the upper bound for Δ.

which implies

$$s = \left(\frac{\ln \left(\frac{\Delta + P_1/P - 1}{1 - P_1/P} \right)}{\Delta} \right)^{\frac{1}{\gamma}}.$$

I assume that the sum of the steady state levels of the firm specific productivity variables is equal to zero:

$$\iota_1 = \iota, \quad \iota_2 = -\iota.$$

Further, the steady state value Z of the economy wide productivity shock is set to one. Hence, the two total factor productivities in the stationary equilibrium are given by $1 + \iota$ and $1 - \iota$ for firm 1 and firm 2 respectively. To calibrate ι, I use the conditions for optimal price setting of the two firms:

$$\frac{P_1}{P} = \frac{1}{s^\gamma x_2} + \frac{W/P}{1 + \iota},$$

$$\frac{P_2}{P} = \frac{1}{s^\gamma x_1} + \frac{W/P}{1 - \iota}.$$

Combining these two equations yields:

$$\frac{1 + \iota}{1 - \iota} = \underbrace{\frac{\frac{P_2}{P} - \frac{1}{s^\gamma x_1}}{\frac{P_1}{P} - \frac{1}{s^\gamma x_2}}}_{:=\varphi}.$$

Thus ι can be computed as:

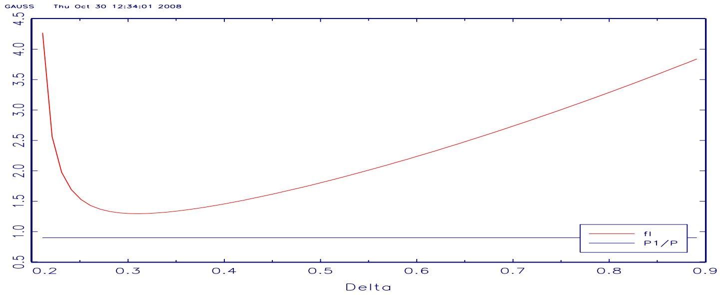
$$\iota = \frac{\varphi - 1}{\varphi + 1}.$$

Note that both the numerator and the denominator of φ must be grater than zero.[17] But since $P_2/P > P_1/P$ and $s^\gamma x_1 > s^\gamma x_2$ it suffices to ensure that the difference $\frac{P_1}{P} - \frac{1}{s^\gamma x_2}$ is positive. This requirement imposes an upper bound on Δ which can be approximated numerically. The numerical analysis[18] also allows us to find a lower bound for P_1/P. The lhs and the rhs of the inequality

$$\frac{P_1}{P} > \underbrace{\frac{1}{s^\gamma x_2}}_{:=f_l} \tag{5.0.20}$$

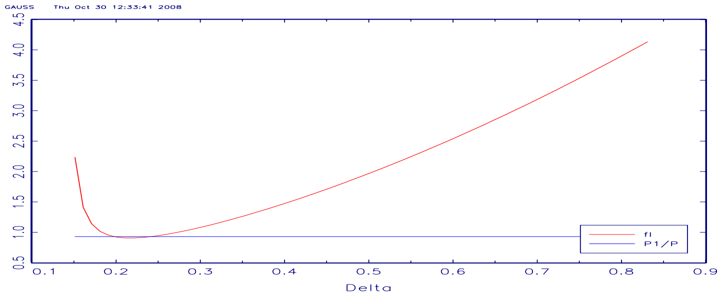
are depicted in figures v.3, v.4 and v.5 for different values of P_1/P and Δ. Recall that s as well as x_2 depend on Δ.

Figure v.3: Lhs of inequality (5.0.20) denoted by f_l. $\frac{P_1}{P} = 0.90$.



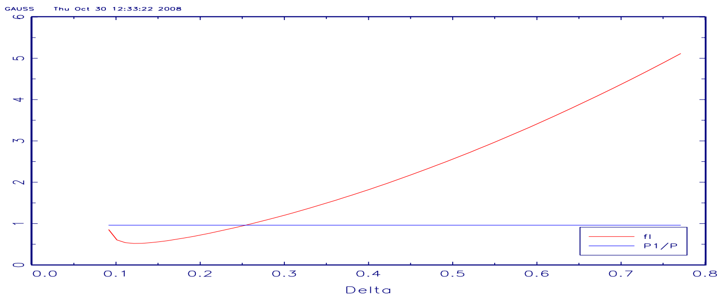
f_l - lhs of equation (5.0.20), *Delta* - Δ (degree of price dispersion).

Figure v.4: Lhs of inequality (5.0.20) denoted by f_l. $\frac{P_1}{P} = 0.93$.



f_l - lhs of equation (5.0.20), *Delta* - Δ (degree of price dispersion).

Figure v.5: Lhs of inequality (5.0.20) denoted by f_l. $\frac{P_1}{P} = 0.96$.



f_l - lhs of equation (5.0.20), *Delta* - Δ (degree of price dispersion).

As long as P_1/P is smaller than (or equal to) 0.92 the term $\frac{1}{s^{\gamma} x_2}$ will be larger than P_1/P irrespective of the measure of price dispersion Δ. If P_1/P takes values in the interval (0.92,0.95), there will be Δs satisfying (5.0.20) but their range will be very small (see figure v.4). In other words, for $P_1/P \in (0.92, 0.95)$ inequality (5.0.20) does not only define an upper but also a lower bound for Δ. The latter is even more restrictive than the one implied by (5.0.19). Accordingly, one needs a sufficiently large P_1/P in order for (5.0.20) to be as unrestrictive as possible with respect to the range of Δ. In particular, for $P_1/P \geq 0.96$ equation (5.0.20) only adds an upper bound for Δ to the restriction defined in (5.0.19). Provided that $\frac{P_1}{P} - \frac{1}{s^{\gamma} x_2} > 0$ is satisfied, it is easy to show that φ is greater than one. $\varphi > 1$ then implies that ι lies in the interval between zero and one.

[17] Otherwise marginal costs will have to be negative.

[18] The corresponding program is "**equilibrim_2a.g**".

An important question regarding the calibration of Δ is to what extent its range is consistent with the empirically observable price dispersion among homogeneous nondurable goods. For example $P_1/P = 0.97$ implies that Δ should range between 0.06 and 0.25, corresponding to a percentage difference between the two prices lying between 6 and 23 percent of P_1/P. Lach (2002) provides evidence on price dispersion for virtually homogeneous commodities based on a panel of stores in the USA. After controlling for observable as well as unobservable sources of heterogeneity[19] between physically homogeneous products he obtains the following estimates: The difference between the logarithms of the 95% and the 5% quantiles of the price distribution equals 0.10, 0.23, 0.22 and 0.16 for *Refrigerator*,[20] *Chicken*, *Coffee* and *Flour* respectively. The differences between the logs of the 75% and the 25% quantiles of the price distributions of the same commodities are equal to 0.03, 0.09, 0.05 and 0.04 respectively. Thus, choosing Δ to imply that $\ln(P_2/P) - \ln(P_1/P)$ lies between 0.10 and 0.20 can be seen as a compromise calibration. For the sake of completeness I also perform a sensitivity analysis with respect to Δ.

Next, by using the first order condition for optimal price setting of one of the firms one can compute the steady state value of the real wage:

$$\frac{W}{P} = (1+\iota)\left(\frac{P_1}{P} - \frac{1}{s^{\gamma} x_2}\right).$$

To calibrate the velocity of money with respect to output $v_y = Y\frac{\pi}{m}$ I use national accounts data provided by the NIPA. Real balances $\frac{M_t}{P_{t-1}}$ are measured as the ratio of the monetary aggregate M1 divided by the value of the nondurables consumption deflator in the previous period (base year 2000). The same deflator is also used to compute the inflation factor. Aggregate output is measured by the gross national product at constant prices (base year 2000) adjusted by the imputed product generated by the stock of durable goods and the government capital stock.[21] The mean of the velocity of money for the period from 1973:Q1 through 2003:Q4 equals 2.15. Given v_y one is able to compute the parameter a *via* the condition governing the optimal level of search efforts (3.1.11) evaluated at the symmetric steady state:

$$\frac{\alpha}{a\gamma} s^{\alpha-\gamma} = \left(\sum_{l=1}^{2} x_{l,t}\left(1 - \frac{P_l}{P}\right)^2\right)\underbrace{\frac{\pi}{m}Y}_{:=v_y}.$$

The fraction of time spent working in the stationary equilibrium N is set to 0.1386. The labor inputs of the two firms, N_1 and N_2, can be computed as the solution to the following

[19] Lach (2002) controls for the store selling the particular product, the location of the store, the type of the store as well as for time effects.

[20] Note that refrigerators are durable goods.

[21] The computation procedure is described in Cooley and Prescott (1995).

system of equations:

$$\frac{N_1}{N_2} = \overbrace{\frac{x_1}{x_2}\frac{(1-\iota)}{(1+\iota)}}^{:=\varpi}$$
$$N_1 + N_2 = N.$$

The result reads:

$$N_1 = \frac{\varpi}{1+\varpi}N, \qquad N_2 = \frac{1}{1+\varpi}N.$$

Then the steady state levels of output and consumption are readily computed *via*:

$$Y = \frac{P_1}{P}(1+\iota)N_1 + \frac{P_2}{P}(1-\iota)N_2,$$

$$\frac{m}{\pi} = \frac{Y}{v_y},$$

$$C = Y - g(s)\frac{m}{\pi}.$$

The first of these equations reflects the definition of the economy's national product: the latter equals the sum of the two firm-specific products, both measured in units of a common numeraire. The second equation reflects the definition of the velocity of money with respect to output. The third equation stems from the requirement that in equilibrium aggregate supply Y equals aggregate demand $D = C + g(s)\frac{m}{\pi}$.

The remaining parameters are set to the so called *standard values* usually found in the literature. Table 5.1 summarizes the calibration of the model.

Table 5.1:
Calibration

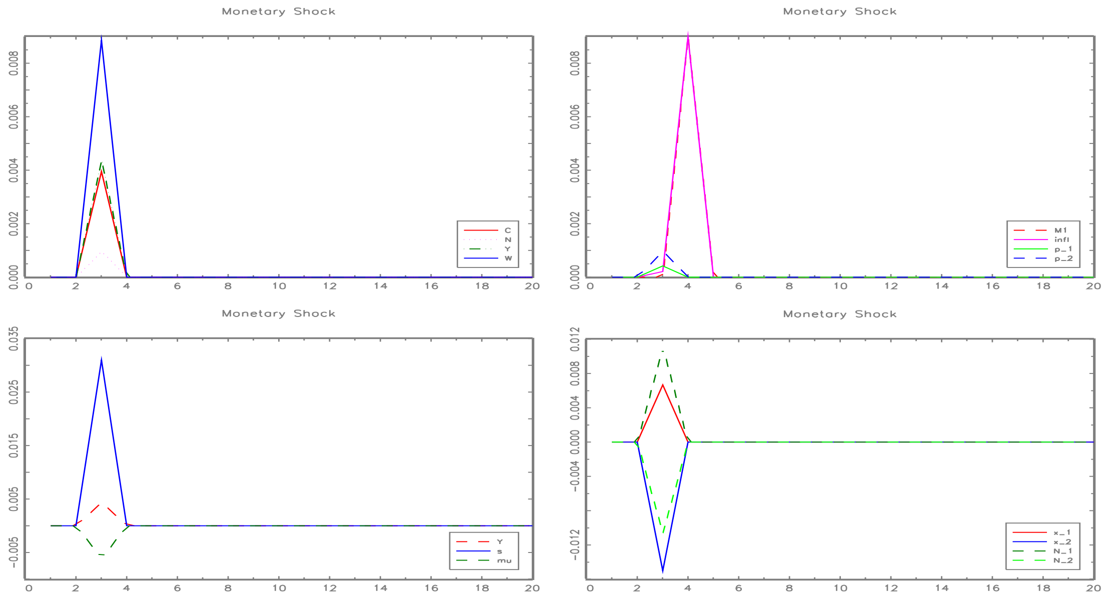
Households/Preferences	Firms/Technology	Central Bank
$\alpha > 0$ sensitivity analysis	$Z^* = 1$	$\tau^* = 1.0138$
$\gamma > 0$ sensitivity analysis	$\rho_z = \{0, 0.9641\}$	$\rho_\tau = 0$
$\beta = 0.991$	$\sigma_\epsilon = 0.0082$	$\sigma_u = 0.0092$
$\eta = 2$	$P_1/P \in (0.95, 0.99]$	$v_y = 2.15$
$\chi = 2$	Δ sensitivity analysis	

6 Results

6.1 Monetary Shocks

Figures vi.6 through vi.9 depict the impulse responses to a one time monetary expansion in the third period, computed with different sets of parameters.[22] As can be seen, such reparameterizations affect the quantitative predictions of the model but leave its qualitative properties almost unaffected. In particular, the larger difference between α and γ and/or the larger the degree of price dispersion Δ, and/or the higher the relative price[23] P_1/P, the lower the magnitude of the reactions to the monetary disturbance. At the same time, the difference $\alpha - \gamma$ ought to be sufficiently large in order to ensure the local uniqueness of the equilibrium.[24] The critical value of $\alpha - \gamma$, below which there are multiple equilibria, depends on the other parameters of the model.

Figure vi.6: Impulse responses to a monetary shock, $\rho_{\tau} = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.12$, $P_1/P = 0.97$.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

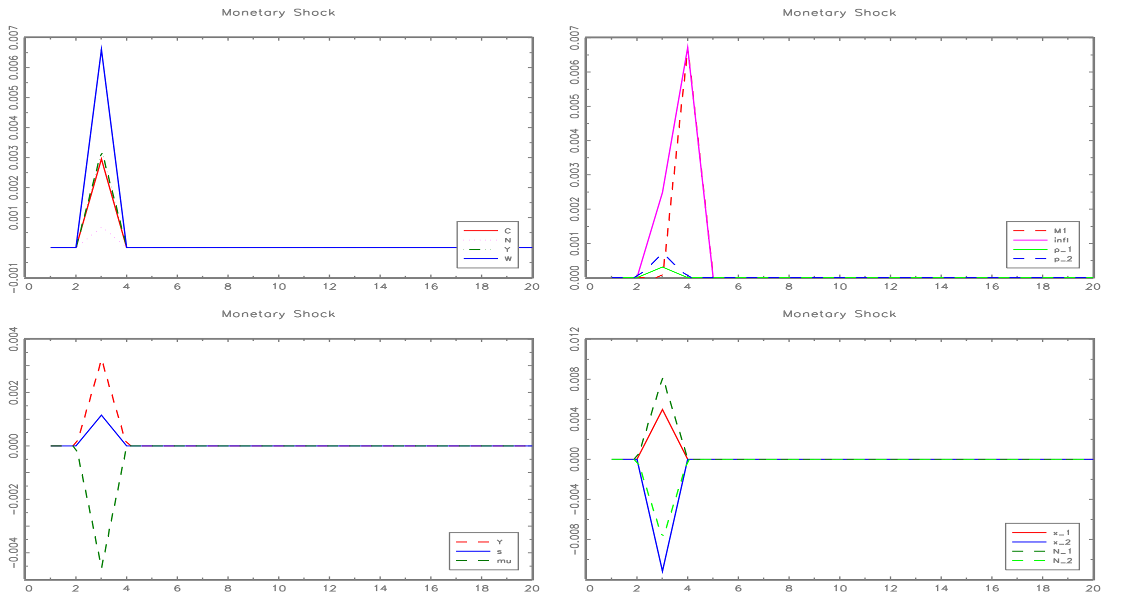
What's the intuition behind these results? For a given price level the monetary expansion induces a positive income effect which forces households to consume more and work less. As a consequence, there is a huge positive pressure on nominal prices and inflation. In a standard model with fully flexible prices and additively separable utility function the increase in the inflation rate will be just sufficient to offset the positive income effect of the monetary shock. In contrast, in the economy presented in this paper a higher desired level of consumption increases the benefit of additional search efforts. At the same time the higher inflation

[22] The corresponding program is "**sim_cm2d6a.g**".

[23] Recall that P_1/P is the relative price of the firm charging the lower price.

[24] Otherwise, the log-linear version of the model has too many eigenvalues inside the unit circle.

Figure vi.7: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 20$, $\gamma = 14$, $\Delta = 0.12$, $P_1/P = 0.97$.



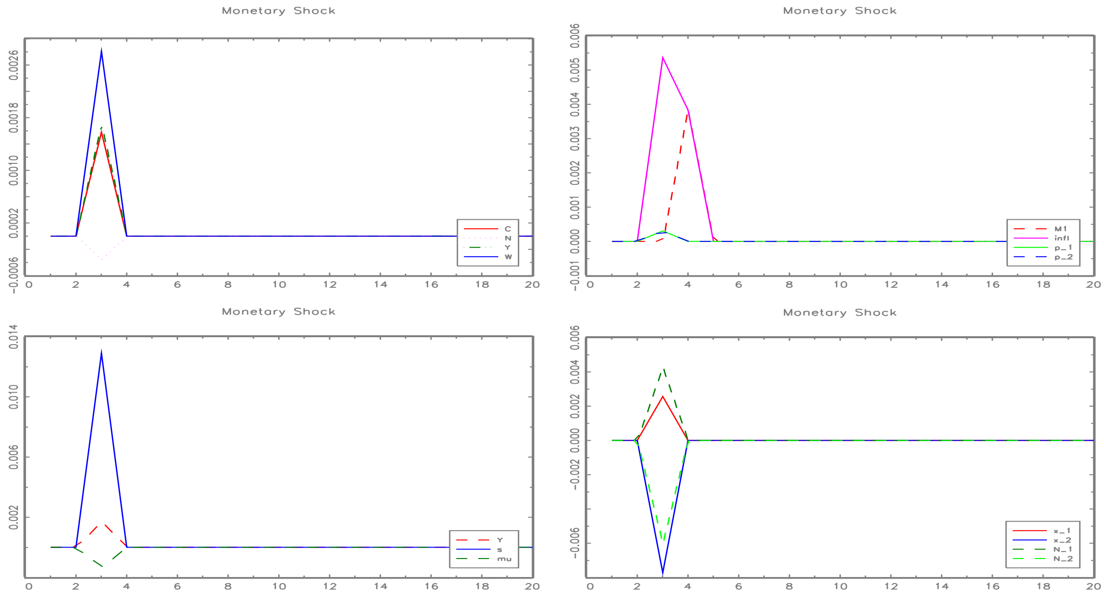
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

reduces the transaction costs per unit of search. Both effects create an incentive for agents to increase their search activity in the goods market. In an environment characterized by a more intense search each firm, fearing a decline in its market share, will be reluctant to pass through to prices the whole increase in marginal costs. Hence, there will be a fall in markups, leading to an increase in real wages. The latter effect induces households to work more which, in turn, enables the economy to produce more and dampens the positive pressure on current inflation. Indeed, if the difference between α and γ is not too large, there is virtually no reaction of inflation in the period of the shock (see for example figure vi.6). In all simulations performed the monetary expansion leads to a drop in both firm-specific markups. However, depending on the calibration chosen, the reaction of firm 1's markup can be stronger or weaker than that of firm 2's one. Note that since search activity as well as the market share of firm 1 increase, there is an unambiguous negative effect on the markup of firm 2. In the case of firm 1, however, there are two effects working in opposite directions: while the increase in search efforts reduces firm 1's markup $mu_{1,t}$, the decline in the other firm's market share, x_2 has a positive effect on $mu_{1,t}$. As figures vi.6 through vi.9 show, the rise in search activity also leads to a large shift of demand towards the cheaper supplier, reflected by the healthy increase in his market share x_1.[25]

According to vi.6 through vi.9, the real effects of the monetary shock disappear after one period. The reason for this absence of persistence is that in the period after the shock the increase in real balances is exactly offset by an equally strong reaction of inflation. Hence, in

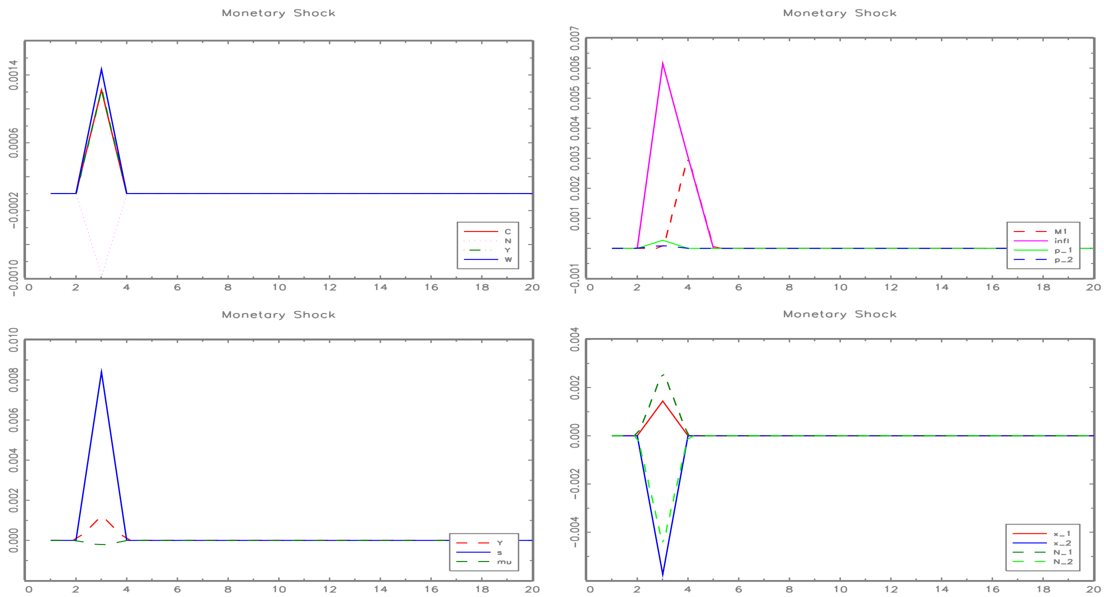
[25] See the fourth panel in figures vi.6 through vi.9.

Figure vi.8: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.16$, $P_1/P = 0.97$.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure vi.9: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.20$, $P_1/P = 0.97$.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

the period after the shock there are neither any positive wealth effects on consumption and leisure nor any positive (or negative) pressure on search activity *via* the term $\frac{m_t}{\pi_t}$. Thus, there are no incentives for agents to search more (or less) than in the stationary equilibrium.

A further interesting feature of the model is that as a reaction to a monetary expansion both relative prices deviate positively from their respective steady state values. However, there is no violation of the definition of the overall price index since the impulse responses of P_1/P and P_2/P are accompanied by suitable reactions of the market shares x_1 and x_2.

Summarizing the results, I would like to point out that even though the model presented here does not reproduce the empirically observable shape and persistence of the impulse responses to monetary shocks, it proposes a simple mechanism which substantially amplifies the real effects of monetary policy, making their magnitude consistent with the empirical estimates. Furthermore, as figure vi.6 suggests, combining the current model with other theoretical building blocks could be a fruitful line of research when trying to explain the observable *delayed* response of inflation to monetary expansions.

6.2 Technology Shocks

Figures vi.10 through vi.12 depict the impulse responses to an economy wide technology shock with no serial correlation. In all cases there is a slight decrease in both relative prices, whereas the reaction of P_2/P is stronger. There is a large drop in search activity and perhaps surprisingly, an increase in the market share of the firm charging the higher price. The intuition behind these results is as follows: The technology shock enables the economy to produce more even by employing less labor and so puts a downward pressure on inflation. The fall in inflation has a strong negative effect on search activity. The lower level of search intensity in the goods market enables both firms to choose higher markups. At the same time, due to the technological improvement, for any given real wage marginal costs become lower. The firm with the lower steady state productivity (firm 2) faces a larger marginal cost decrease, which enables it to reduce its relative price P_2/P by a larger percentage amount than firm 1 does.[26] The stronger (weaker) decrease in the relative price of firm 2 (firm 1) combined with a lower search activity in the goods market, in turn, leads to an increase in the market share of the supplier charging the higher price (firm 2).

What happens to markups? The fall in both, overall search efforts and the market share of firm 1, unambiguously makes it possible for firm 2 to increase its markup. With regard to firm 1 there are again two opposing effects: The decline in search intensity allows firm 1 to set a higher markup but the increase in the market share of its competitor does the opposite. Which of these two effects dominates depends on the value of γ. To see this, just log-linearize the first order condition for optimal price setting of firm 1 and take a look at

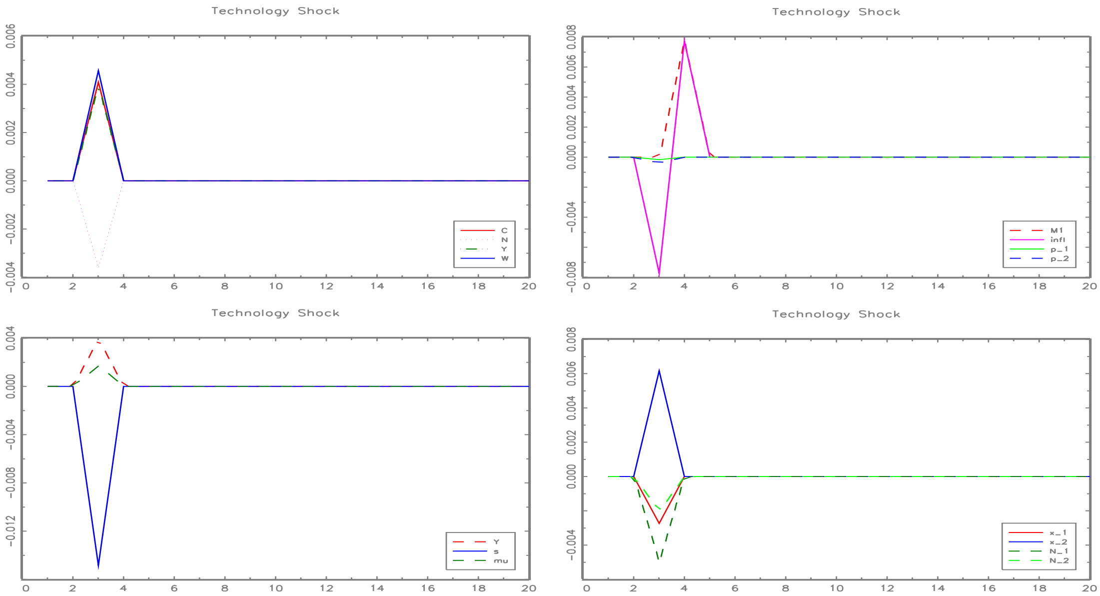
[26]The direct effect of the productivity disturbance on the relative price is measured by the term $\frac{W/P}{(P_1/P)(1+\iota)}\hat{Z}_t$ in the case of firm 1 and by $\frac{W/P}{(P_2/P)(1-\iota)}\hat{Z}_t$ for firm 2. In all numerical simulations performed $(P_1/P)(1+\iota)$ is larger than one while $(P_2/P)(1-\iota)$ takes a value below one.

the elasticities in front of s_t and $x_{2,t}$. The log-linear equation reads:

$$\widehat{\left(\frac{P_{1,t}}{P_t}\right)} = -\frac{\gamma}{(P_1/P)s^{\gamma}x_2}\hat{s}_t - \frac{1}{(P_1/P)s^{\gamma}x_2}\hat{x}_{2,t} + \frac{W/P}{(P_1/P)(1+\iota)}\widehat{\left(\frac{W_t}{P_t}\right)} - \frac{W/P}{(P_1/P)(1+\iota)}\hat{Z}_t.$$

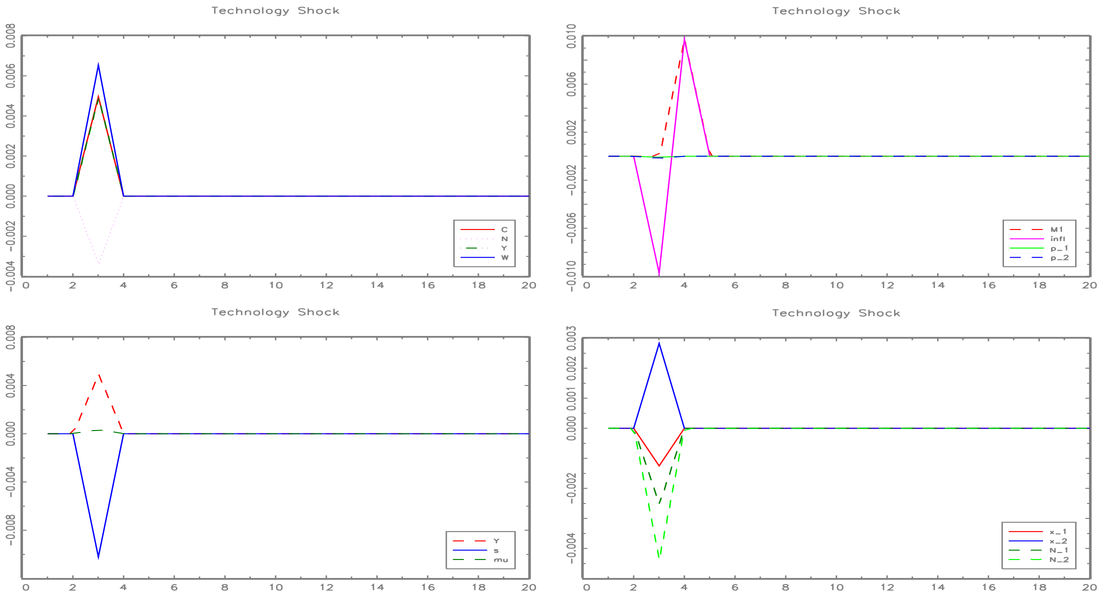
Now it is readily seen that the smaller the parameter γ, the lower the value of the elasticity $\frac{\gamma}{(P_1/P)s^{\gamma}x_2}$ and thus, the smaller the importance of changes in search activity for the determination of $P_{1,t}/P_t$. Hence, if γ is sufficiently low, firm 1 will reduce its markup as a reaction to a positive productivity shock. The latter is supported by figures vi.13 through vi.15 which show the impulse responses of the firm-specific markups for different values of γ. The other sets of parameters examined reveal qualitatively the same picture. Furthermore, almost all of the paramtereizations used imply that the average markup is countercyclical.

Figure vi.10: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.6$, $\Delta = 0.12$, $P_1/P = 0.97$.



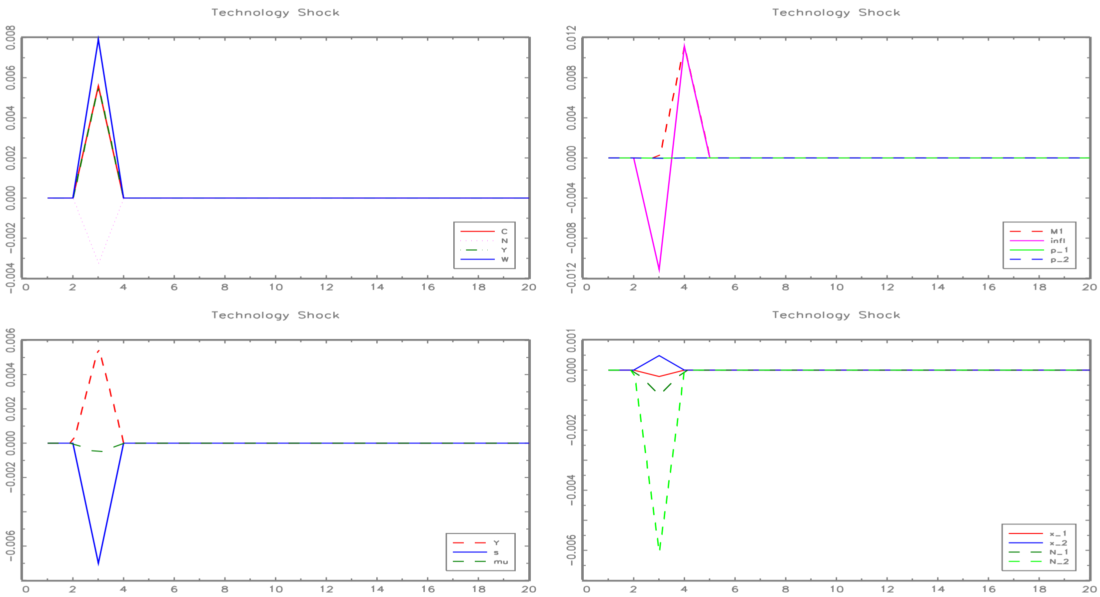
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure vi.11: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.4$, $\Delta = 0.12$, $P_1/P = 0.97$.



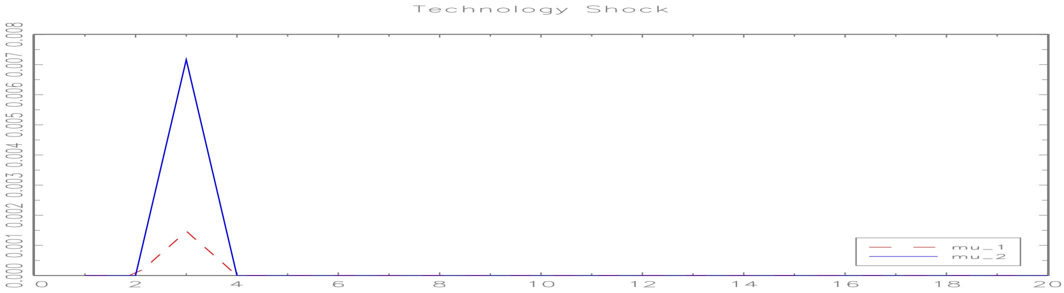
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure vi.12: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.1$, $\Delta = 0.12$, $P_1/P = 0.97$.



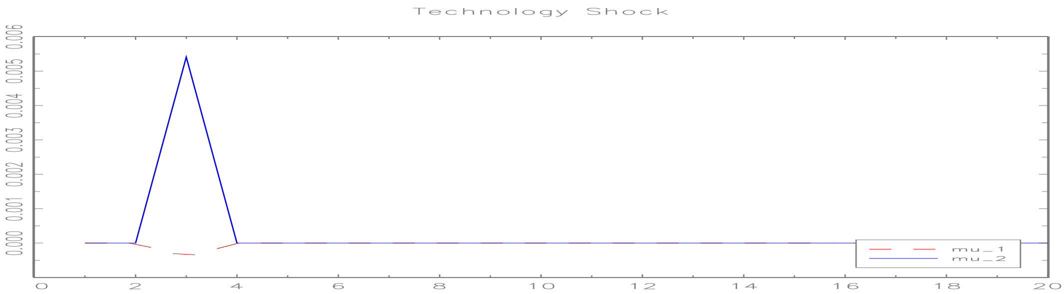
Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure vi.13: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.1$, $\Delta = 0.12$, $P_1/P = 0.97$.



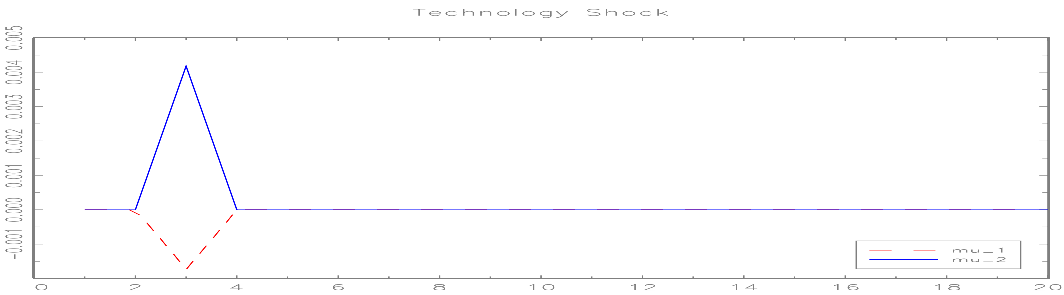
mu_1 - markup of firm 1, mu_2 - markup of firm 2.

Figure vi.14: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.6$, $\Delta = 0.12$, $P_1/P = 0.97$.



mu_1 - markup of firm 1, mu_2 - markup of firm 2.

Figure vi.15: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.4$, $\Delta = 0.12$, $P_1/P = 0.97$.



mu_1 - markup of firm 1, mu_2 - markup of firm 2.

7 Capital Accumulation

7.1 The Model

Let us extend the model by assuming that there are two production factors - capital and labor. The production function of firm l exhibits constant returns to scale and is given by

$$Y_{l,t} = (Z_t + \iota_{l,t}) N_{l,t}^{\omega} K_{l,t}^{1-\omega}, \qquad \omega \in (0,1),$$

where $K_{l,t}$ denotes capital input and $Z_t + \iota_{l,t}$ represents total factor productivity following the same stochastic process as in section 3. The aggregate stock of capital evolves according to

$$K_{t+1} = I_t + (1-\upsilon) K_t, \qquad \upsilon \in (0,1). \tag{7.1.1}$$

Marginal costs of firm l, $\mu_{l,t}$ are now given by

$$\mu_{l,t} = \frac{(W_t/P_t)^{\omega} R_t^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(Z_t + \iota_{l,t})}, \tag{7.1.2}$$

where R_t denotes the rental rate of capital.

7.1.1 Household's First Order Conditions

Retaining the notation used in section 3 the set of first order conditions describing the behavior of he typical household have to be extended by the following three equations:

$$\Lambda_t = \beta E_t \{\Lambda_{t+1}(1 + R_{t+1} - \upsilon)\},$$

$$\underbrace{C_t + I_t + g(s_t)\frac{m_t}{\pi_t}}_{:=D_t} + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + R_t K_t + \Pi_t + \frac{T_t}{P_t}, \tag{7.1.3}$$

$$K_{t+1} = I_t + (1-\upsilon) K_t.$$

The first condition is the Euler equation governing optimal capital accumulation. The second one is the modified budget constraint. Note that in this version of the model aggregate demand D_t equals the sum of consumption, investment and the expenditure on transaction services. The last equation in (7.1.3) is the low of motion of the capital stock.

7.1.2 Equilibrium

The evolution of the economy is described by the *new* definition of marginal costs (7.1.2), the first order condition for optimal price setting (3.1.3), the households first order conditions (3.1.7), (3.1.8), (3.1.9) and (3.1.11) modified by (7.1.3), the aggregate consistency conditions

$$x_{l,t}D_t = (Z_t + \iota_{l,t})N_{l,t}^{\omega}K_{l,t}^{1-\omega} \qquad l = \{1, 2, ..., n\}, \tag{7.1.4}$$

and

$$\sum_{l=1}^{n} N_{l,t} = N_t, \qquad \sum_{l=1}^{n} K_{l,t} = K_t \tag{7.1.5}$$

as well as the definitions of $x_{l,t}$ for $l = \{1, 2, ..., n\}$ and D_t. Note that if (8.2.4) are satisfied, then the family's budget constraint implies:

$$D_t = Y_t = \sum_{l=1}^{n} \frac{P_{l,t}}{P_t}(Z_t + \iota_{l,t})N_{l,t}^{\omega}K_{l,t}^{1-\omega}.$$

To close the model, one again needs to specify monetary policy and the exogenous productivity processes Z_t and $\iota_{l,t}$.

7.1.3 Calibration

P_1/P. α, γ, the measure of price dispersion Δ, the velocity of money with respect to output v_y, ι, N and the parameter a are calibrated in the same way as in the fixed capital case.[27][28] Then the marginal costs of the two firms can be calibrated by using their price setting conditions:

$$\mu_1 = \frac{(W/P)^{\omega}R^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(1+\iota)} = \left(\frac{P_1}{P} - \frac{1}{s^{\gamma}x_2}\right), \qquad \mu_2 = \frac{(W/P)^{\omega}R^{1-\omega}}{\omega^{\omega}(1-\omega)^{1-\omega}(1-\iota)} = \left(\frac{P_2}{P} - \frac{1}{s^{\gamma}x_1}\right)$$

To calibrate the production elasticity of labor, first observe that the real wage and the two marginal costs are related as follows:

$$\frac{W}{P} = \mu_1(1+\iota)\omega N_1^{\omega-1}K_1^{1-\omega} = \mu_2(1+\iota)\omega N_2^{\omega-1}K_2^{1-\omega}.$$

Hence,

$$\frac{W}{P}N_1 = \omega\mu_1 \underbrace{(1+\iota)N_1^{\omega}K_1^{1-\omega}}_{=x_1Y}, \qquad \frac{W}{P}N_2 = \omega\mu_2 \underbrace{(1+\iota)N_2^{\omega}K_2^{1-\omega}}_{=x_2Y}.$$

Adding the last two equations together and rearranging yields:

$$\omega = \underbrace{\frac{(W/P)N}{Y}}_{:=\text{ labor share}} \cdot \frac{1}{x_1\mu_1 + x_2\mu_2}.$$

[27] See section 5.

[28] The analysis of the sensitivity of the steady state with respect to Δ can be found in the program "**equilibrium_3.g**".

The labor share is computed on the basis of the national accounts data provided by the NIPA and transformed in the way suggested by Cooley and Prescott (1995). The value obtained equals 0.6748. Unfortunately, $P_1/P < 0.98$ implies that the range of values of Δ consistent with $\omega \in (0,1)$ is extremely small. When the relative price P_1/P is larger or equal to 0.98, Δ should be smaller than 0.09 in order for ω to lie in the range between zero and one.

The consumption-output ratio $\frac{C}{Y}$ is found by using the economy's resource constraint:

$$\frac{C}{Y} = \frac{1 - \frac{g(s)}{v_y}}{1 - \frac{I}{C}},$$

where the investment-consumption ratio $\frac{I}{C}$ is set at its empirical value 0.1982 obtained with date from NIPA's national accounts, transformed as in Cooley and Prescott (1995).

To determine the ratio K_1/K_2 I use the first order conditions for optimal capital input of both firms, evaluated at the stationary equilibrium:

$$\frac{R}{R} = 1 = \frac{\mu_1}{\mu_2}\frac{1+\iota}{1-\iota}\left(\frac{N_1}{N_2}\right)^{\omega}\left(\frac{K_1}{K_2}\right)^{-\omega}. \tag{7.1.6}$$

Multiplying the rhs of the last equation by $\frac{K_1}{K_2} \cdot \frac{K_2}{K_1}$ and rearranging yields:

$$1 = \frac{\mu_1}{\mu_2}\frac{x_1}{x_2}\frac{Y}{Y}\frac{K_2}{K_1}$$

which implies:

$$\frac{K_1/Y}{K_2/Y} = \underbrace{\frac{\mu_1}{\mu_2}\frac{x_1}{x_2}}_{:=\varphi_2}.$$

Then the aggregate consistency condition for capital input

$$\frac{K_1}{Y} + \frac{K_2}{Y} = \frac{K}{Y}$$

allows me to determine K_1/Y and K_2/Y:

$$\frac{K_1}{Y} = \frac{\varphi_2}{1+\varphi_2}\frac{K}{Y}, \qquad \frac{K_2}{Y} = \frac{1}{1+\varphi_2}\frac{K}{Y}.$$

The economy wide capital intensity K/Y is estimated with data provided by the NIPA. The value obtained is 17.44. Having found K_1/Y and K_2/Y one can use equation (7.1.6) and the aggregate consistency condition

$$N_1 + N_2 = N$$

to compute N_1 and N_2. The result is:

$$N_1 = \frac{\varphi_1}{1+\varphi_1}N, \qquad N_2 = \frac{1}{1+\varphi_1}N,$$

where

$$\varphi_1 = \frac{K_1}{K_2}\left(\frac{\mu_1}{\mu_2}\frac{1-\iota}{1+\iota}\right)^{\frac{1}{\omega}}.$$

The depreciation rate υ is calibrated as in the previous sections.

7.2 Results

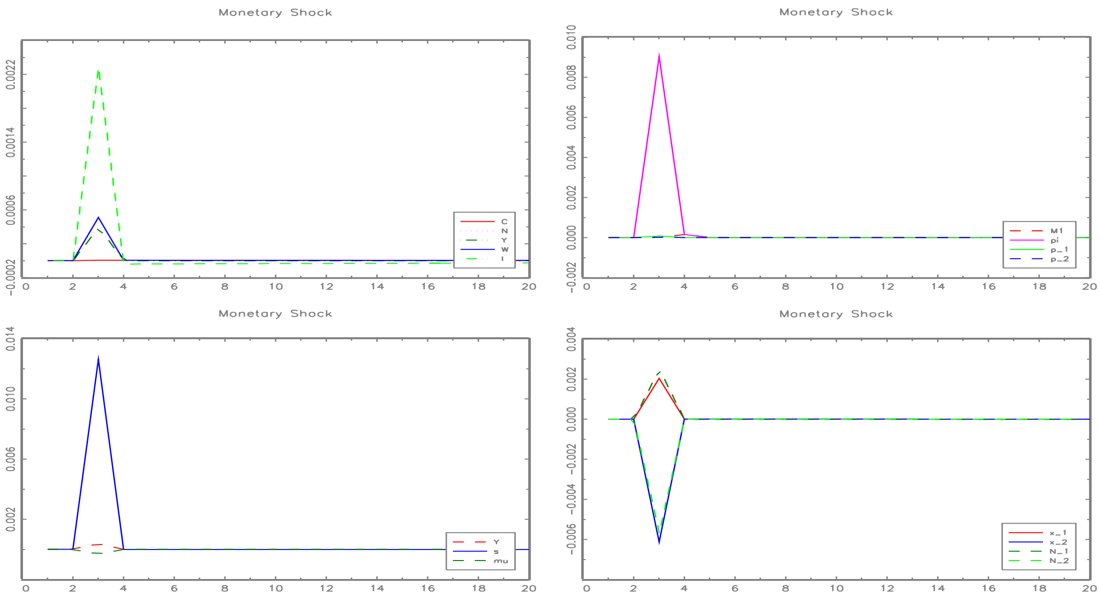
Monetary Shocks: Figure vii.16 depicts the impulse responses to a non-autocorrelated monetary shock.[29] Obviously, the model with flexible capital delivers the same qualitative predictions as the one without capital accumulation: The monetary shock is expansionary mainly due to the fall in markups and triggers off relatively strong reactions of the main economic aggregates. However, these reactions have again the counterfactual property of being extremely short-lived. Again, the larger the difference between α and γ and/or the higher the degree of price dispersion measured by Δ the weaker the real effects of the monetary shock. In general, in this version of the model the response of consumption is of much smaller magnitude than it was the case in the economy presented in section 3. The reason is that capital accumulation allows a more effective consumption smoothing. To take advantage of this possibility, households sharply increase investment in the period of the shock, absorbing in this way virtually the whole additional output. The resulting increase in the capital stock enables the economy to produce and consume more over a relatively long period of time. However, the deviations of output and consumption from their respective steady state values in the periods after the shock are very small.

Technology Shocks: The reactions to technology shocks predicted by the model with flexible capital are also similar to their counterparts implied by the model of section 3 (see figure vii.17). For the bulk of the parameterizations examined markups respond negatively to improvements in productivity. As a result, in almost all cases markups are countercyclical on average. The incentive to smooth consumption over time is again strong enough to force households to substantially accelerate capital accumulation in the period of the shock. As a consequence, virtually the whole additional production is again absorbed by investment expenditure. In the periods after the shock there is a *very small* positive deviation of consumption from its steady state value financed by the additionally accumulated capital.

Unfortunately, the inclusion of capital accumulation does not make the predictions of the model with respect to the persistence of the impulse responses to monetary and real shocks more realistic.

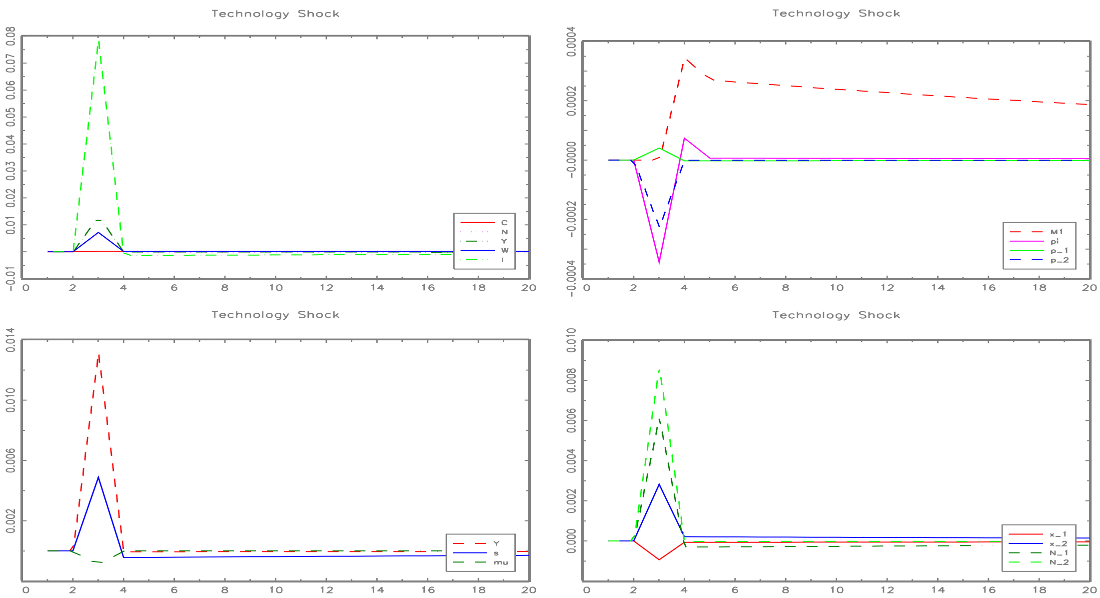
[29] The corresponding program is "**sim_cm2d7a.g**".

Figure vii.16: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.04$, $P_1/P = 0.99$.



Y - output, N - hours, C - consumption, W - real wage, I - investment, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

Figure vii.17: Impulse responses to a technology shock, $\rho_z = 0$, $\alpha = 0.9$, $\gamma = 0.7$, $\Delta = 0.04$, $P_1/P = 0.99$.



Y - output, N - hours, C - consumption, W - real wage, I - investment, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, P_1 - relative price of firm 1, P_2 - relative price of firm 2, s - search activity, mu - average markup, N_1 - labor input of firm 1, N_2 - labor input of firm 2, x_1 - market share of firm 1, x_2 - market share of firm 2.

8 Shopping-Time Models

8.1 A Standard Shopping-Time Model

Consider an economy with fully flexible prices and perfectly competitive markets. The utility function of the representative household takes the form

$$U = E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\eta}}{1-\eta} - \frac{b}{2}(1-L_t)^2 \right) \right\}, \qquad b, \eta > 0, \quad \beta \in (0,1),$$

where L_t denotes leisure. The corresponding budget constraint is given by

$$C_t + \frac{M_{t+1}}{P_t} - \frac{M_t}{\pi_t P_{t-1}} = \frac{W_t}{P_t} N_t + \frac{T_t}{P_t}, \quad \forall t.$$

A positive valuation of money arises through the following *"shopping-time"* constraint:

$$\frac{s_t^\alpha}{1 + s_t^\alpha} = \kappa \frac{C_t}{m_t/\pi_t}, \tag{8.1.1}$$

where $\kappa > 0$ and $\frac{s_t^\alpha}{1+s_t^\alpha}$ is the time needed to carry out transactions in the goods market. According to (8.1.1) a higher real value of the money balances accumulated in the previous period, $\frac{m_t}{\pi_t}$ reduces he transaction time associated with a given desired level of consumption and thus, lowers the transaction costs. The shopping-time technology (8.1.1) originates from the idea that in a typical barter economy each agent faces extremely large search costs since she can only achieve the desired consumption bundle if she is able to find enough other individuals supplying exactly the goods our agent desires and at the same time, demanding exactly the good(s) she supplies. The search costs, however, can be substantially reduced by the introduction of money as a common medium of exchange and unit of account. The transaction cost motive for holding money dates back to Baumol (1952) and Tobin (1956). In more recent papers Saving (1971), Jovanovich (1982) and Romer (1986) develop general equilibrium versions of the shopping-time model. The model analysed in this subsection is very similar to the one presented in Walsh (2003), Ch. 3.

The time constraint of the household reads:

$$L_t + N_t + \frac{s_t^\alpha}{1 + s_t^\alpha} = 1.$$

The representative firm produces according to the production function:

$$Y_t = Z_t N_t,$$

where Z_t evolves according to the same stochastic process as in the previous sections. The behavior of the central bank, too, is modeled as in section 3.

Impulse Responses: As figure viii.18 shows the real effects of a one-time increase in money supply are negligible.[30] [31] Furthermore, the monetary expansion is *contractionary*. It leads to a decline of working hours, output and consumption, while the real wage remains unchanged.[32] The time spent shopping is the only variable which reacts positively to the monetary shock. In addition, the model os not able to reproduce the persistence observable in the data. Variations

[30] The remaining parameters are set at their *standard values*.

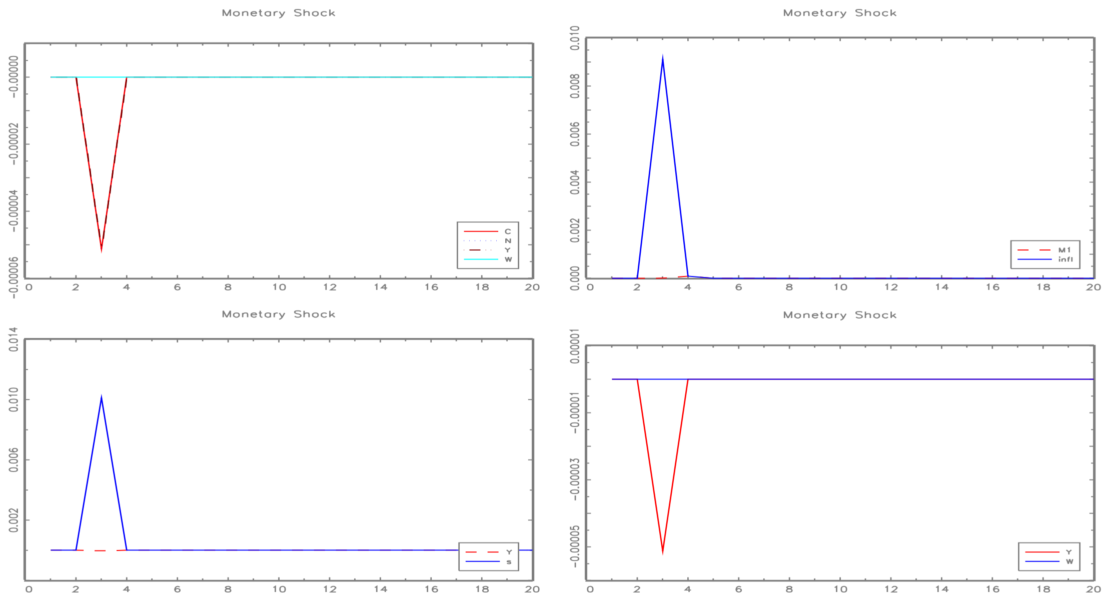
[31] The corresponding program is "**sim_cm2d8c.g**".

[32] Real wages in this economy can only change if total factor productivity changes.

of the parameter α have a negligible effect on the quantitative implications of the model, with higher values of α making the real effects of the monetary disturbance even weaker.

How does the introduction of the kind of market share competition proposed in section 3 alter the predictions of the shopping-time model?

Figure viii.18: Impulse responses to a monetary shock, $\rho_T = 0$, $\alpha = 0.9$.



Y - output, N - hours, C - consumption, W - real wage, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, s - measure of transaction time.

8.2 Shopping-Time and Market Share Competition I

Let us assume that the structure of the goods market is the same as the one described in section 3. Assume further that the representative household solves the same problem as in subsection 8.1 but the fraction of time spent shopping affects the individual market shares of the firms in this economy. Assume that the household does not internalize this effect. The latter can be thus, characterized as an externality induced by shopping or as a by-product of shopping. The idea behind this assumption is that the more time households spend shopping and thus, the more transactions they are involved in, the better informed they are about the current price distribution as well as the price setting behavior of the individual suppliers. This information allows households to at least pasrtly shift their towards the relatively cheap suppliers.

The market share of firm l evolves according to:

$$x_{l,t} = \frac{\exp\left(\left(1 - \frac{P_{l,t}}{P_t}\right) s_t^{\gamma}\right)}{\sum_{j=1}^{n} \exp\left(\left(1 - \frac{P_{j,t}}{P_t}\right) s_t^{\gamma}\right)}, \qquad \gamma > 0. \tag{8.2.1}$$

Hence, firms with above average prices suffer larger losses in market share when households devote more time to transactions in the goods market. Note that in this model aggregate demand equals aggregate consumption expenditure.

Equilibrium: The evolution of this economy is described by the following set of equations. The utility maximization problem of the representative household delivers the following first order conditions:

$$
\begin{aligned}
&C_t^{-\eta} = \Lambda_t + \kappa \frac{\Gamma_t \pi_t}{m_t}, \\
&b(1 - L_t) = \Lambda_t \frac{W_t}{P_t}, \\
&\Lambda_t \frac{W_t}{P_t} = \Gamma \frac{\alpha s_t^{\alpha-1}}{(1 + s_t^{\alpha})^2}, \\
&\frac{s_t^{\alpha}}{1 + s_t^{\alpha}} = \kappa \frac{C_t}{m_t/\pi_t}, \\
&\Lambda_t = \beta E_t \left\{ \kappa \frac{\Gamma_{t+1}}{m_{t+1}^2} C_{t+1} \pi_{t+1} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right\}, \\
&C_t + m_{t+1} - \frac{m_t}{\pi_t} = \frac{W_t}{P_t} N_t + \Pi_t + \frac{T_t}{P_t}, \\
&N_t + \frac{s_t^{\alpha}}{1 + s_t^{\alpha}} + L_t = 1,
\end{aligned}
\tag{8.2.2}
$$

where Λ_t and Γ_t are the lagrangean multipliers associated with the budget constraint and the shopping-time constraint respectively.

The conditions for optimal price setting of the two firms are again given by:

$$
\begin{aligned}
&\frac{P_{1,t}}{P_t} = \frac{1}{s_t^{\gamma} x_{2,t}} + \frac{W_t/P_t}{Z_t + \iota_{1,t}}, \\
&\frac{P_{2,t}}{P_t} = \frac{1}{s_t^{\gamma} x_{1,t}} + \frac{W_t/P_t}{Z_t + \iota_{2,t}}.
\end{aligned}
\tag{8.2.3}
$$

Furthermore, the following aggregate consistency conditions hold:[33]

$$x_{l,t} D_t = (Z_t + \iota_{l,t}) N_{l,t}, \qquad l = \{1, 2\}, \tag{8.2.4}$$

and

$$\sum_{l=1}^{2} N_{l,t} = N_t. \tag{8.2.5}$$

Impulse Responses: Unfortunately, it turns out that there is only a relatively small range of parameter values implying an economically meaningful stationary equilibrium.[34] In particular, to ensure that the steady state value of leisure is positive, the relative price P_1/P should be lower than 0.96 and at the same time, the difference $\alpha - \gamma$ should be sufficiently low. The critical value for $\alpha - \gamma$ depends on the absolute values of these parameters as well as on P_1/P. Figure viii.19 depicts the impulse responses to a monetary disturbance without serial correlation. The increase in inflation necessitates a higher level of transaction time. As a result firms are forced to reduce their markups which in turn, leads to a real wage increase. However, the latter is not sufficient to induce households to work more and thus, the economy to produce more. The reason is that the jump in inflation generates too strong an incentive for agents to raise shopping time, so that labor supply and leisure ought to be reduced. As a consequence, the positive monetary shock again leads to an economic contraction in which the real wage and the time spent shopping are the only non-nominal aggregates deviating positively from their respective steady state values.

As can also be seen, the real effects of monetary policy are much more pronounced in this version of the shopping-time model than in the one described in subsection 8.1. The sensitivity analysis performed revealed that varying the model parameters within the economically meaningful range has a negligible effect on the quantitative predictions of the model.

8.3 Shopping-Time and Market Share Competition II

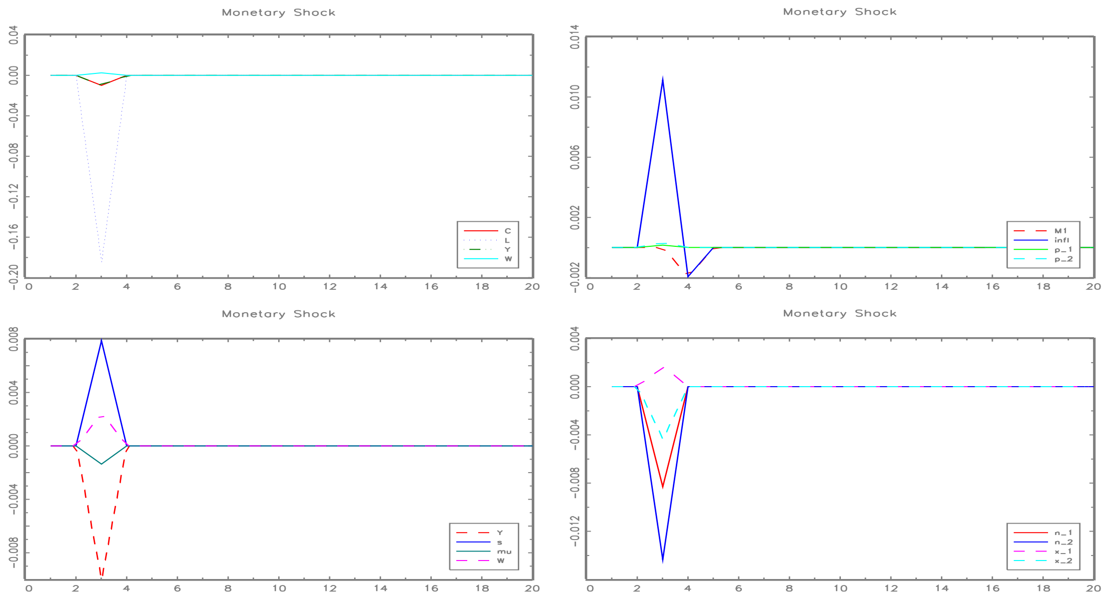
Now assume that households are aware of the link between the time they spent carrying out transactions and the probability to become a customer of a particular firm. Assume further that the household sector has the structure proposed in section 3, so that the utility

[33] The overall price index is defined as:

$$\sum_{l=1}^{n} x_{l,t} \frac{P_{l,t}}{P_t} = 1.$$

[34] The corresponding programs are "**equilibrium_4.g**" for the analysis of the steady state and "**sim_cm2d8b.g**" for the computation of the impulse responses provided in this paragraph.

Figure viii.19: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.8$, $P_1/P = 0.94$, $\Delta = 0.18$.



Y - output, N - hours, C - consumption, W - real wage, L - leisure, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, p_1 - relative price of firm 1, p_2 - relative price of firm 2, s - measure of transaction time, mu - average markup, x_1 - market share of firm 1, x_2 - market share of firm 2, n_1 - labor input of firm 1, n_2 - labor input of firm 2.

maximization is performed in two steps and the equilibrium is symmetric. In the second step of utility maximization, given the lagrangeans Λ_t and Γ_t, the real wage W_t/P_t and the level of consumption expenditure C_t the typical member of an arbitrary family chooses the optimal level of shopping time. The latter is set according to:

$$\Lambda_t \frac{W_t}{P_t} - \Gamma \frac{\alpha s_t^{\alpha-1}}{(1+s_t^\alpha)^2} = \frac{s_t^{\gamma-1}}{\gamma} \left(\sum_{l=1}^{n} x_{l,t} \left(1 - \frac{P_{l,t}}{P_t}\right)^2 \right) C_t.$$

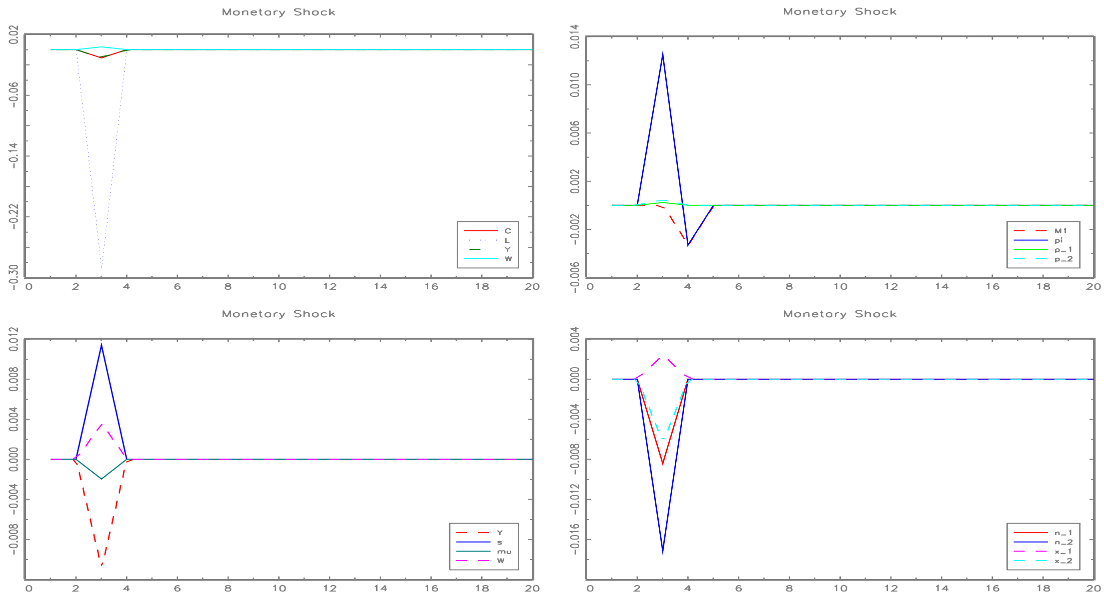
Figure viii.20 displays the impulse responses to the same one-time monetary shock.[35] It is easily seen, that the reactions implied by the modified model are virtually identical to that predicted by the previous version of the model. Thus, the more complicated and more sophisticated theoretical mechanisms underlying the theoretical framework of the current subsection do not eliminate the major weaknesses of the model presented in subsection 8.2.

8.4 Steady State and Calibration

The purpose of this subsection is to summarize the most important calibration steps regarding the models presented in subsections 8.2 and 8.3.

[35] The corresponding program is "**sim_cm2d8a.g**".

Figure viii.20: Impulse responses to a monetary shock, $\rho_\tau = 0$, $\alpha = 0.9$, $\gamma = 0.8$, $P_1/P = 0.94$, $\Delta = 0.18$.



Y - output, N - hours, C - consumption, W - real wage, L - leisure, $M1 = \frac{M_t}{P_{t-1}}$ - real balances, $Infl$ - inflation, p_1 - relative price of firm 1, p_2 - relative price of firm 2, s - measure of transaction time, mu - average markup, x_1 - market share of firm 1, x_2 - market share of firm 2, n_1 - labor input of firm 1, n_2 - labor input of firm 2.

The households' first order conditions with respect to s_t, M_{t+1} and C_t evaluated at the steady state imply:

$$C^{-\eta} = \Lambda + \kappa \frac{\Gamma \pi}{m},$$

$$\Lambda \frac{W}{P} = \Gamma \frac{\alpha s^{\alpha-1}}{(1+s^\alpha)^2} + \underbrace{\frac{s^{\gamma-1}}{\gamma} \left(\sum_{l=1}^{2} x_l \left(1 - \frac{P_l}{P}\right)^2 \right) C}_{:=\xi_5}, \qquad (8.4.1)$$

$$\frac{\pi - \beta}{\pi \beta} = \kappa \frac{C \pi}{m} \frac{\Gamma}{\Lambda} \frac{1}{m}.$$

ξ_5, s and $C = \frac{P_1}{P}(1+\iota)N_1 + \frac{P_2}{P}(1-\iota)N_2$ are calibrated in the same way as in section 5. [36] With respect to α I perform a sensitivity analysis. Then take into account the definition of the shopping-time technology, multiply both sides of the first equation in 8.4.1 by C and

[36] s is given by:

$$s = \left(\frac{\ln\left(\frac{\Delta + P_1/P - 1}{1 - P_1/P}\right)}{\Delta} \right)^{\frac{1}{\gamma}}.$$

solve the resulting system for Λ, $\mathit{\Gamma}$ and m. The *relevant* results are:

$$\Lambda = \frac{\xi_5 + \frac{\alpha}{s(1+s^\alpha)} C^{1-\eta}}{\frac{W}{P} + \frac{\alpha}{s(1+s^\alpha)} C},$$

$$\mathit{\Gamma} = \frac{1+s^\alpha}{s^\alpha}(C^{1-\eta} - \Lambda C), \tag{8.4.2}$$

$$m = \frac{s^\alpha}{1+s^\alpha}\frac{\beta\pi}{\pi-\beta}\frac{\mathit{\Gamma}}{\Lambda}.$$

The steady state value of leisure L is determined by the time constraint with $N = 0.1386$:

$$L = 1 - N - \frac{s^\alpha}{1+s^\alpha}.$$

The calibration of the model presented in subsection 8.2 is obtained by setting ξ_5 equal to zero.

9 Conclusion

The current paper develops a model economy characterized by heterogeneity and price dispersion across firms. The price dispersion, in turn, generates an incentive for households to make search efforts. The latter lead to transaction costs which can be only financed by real balances accumulated in the previous period. At the same time, a more intense search increases the probability for becoming a customer of a supplier charging relatively low prices. The current paper examines the ability of this modified transactions channel to transmit nominal disturbances to the real economy as well as to make the reactions to monetary surprises (more) consistent with the empirical evidence.

The impulse responses implied by the model have the sign predicted by the bulk of the SVAR literature. Unfortunately however, their persistence is not consistent with the empirical evidence.

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