The Cash-In-Advance Constraint in Monetary Growth Models with Labor Market Search*

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Abstract

We study the sensitivity of the inflation-growth trade-off in monetary growth models with respect to the introduction of search frictions in the labor market. We consider three types of endogenous growth models: 1) the AK model, 2) the Lucas (1990) supply-side model, and 3) the two-sector model of Jones and Manuelli (1995). We show that the effects depend on the specification of the cash-in-advance constraint and the magnitude of the semi-interest elasticity of the income velocity. For the AK model, economic growth increases with higher inflation. For the other two models, growth declines for the case of the standard cash-in-advance constraint on consumption, while it either increases or decreases if money is introduced as in the cash-credit good economy of Dotsey and Ireland (1996) depending on the semi-interest elasticity of the income velocity. The welfare effects of inflation are shown to be economically significant in the presence of search unemployment.

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1 Introduction

Frictions in the labor market are central to the understanding of the monetary facts of the business cycle and have already been emphasized in early work, including Keynes (1936) and Phillips (1958). In this tradition, a vast modern literature of New-Keynesian models has evolved that relies on search and matching frictions following Mortenson and Pissarides (1994). Among others, labor market frictions have been found particularly useful in explaining the persistent and hump-shaped response of output to a monetary policy shock, e.g., in Christiano et al. (2005), Walsh (2005), Trigari (2009), and Heer and Maußner (2010), and the inflation dynamics, e.g., in Christoffel et al. (2009a, 2009b). While the short-run business-cycle effects of monetary policy and the inflation dynamics have been studied extensively in these Dynamic Stochastic General Equilibrium models, the long-run effects of inflation on economic growth in the presence of labor market frictions have not received any attention yet. The present article is intended to fill this gap.

Inflation and economic growth are a central subject in the literature, both on growth and monetary economics. Most empirical cross-country studies support the fact that inflation has a negative effect on growth.[1] However, this evidence is much less clear-cut for countries characterized by low inflation so that inflation appears to have a non-linear effect on growth. Barro (1996) and Gillman et al. (2004) point out that the latter effect is not robust if the endogeneity problem is accounted for. In a recent study using cross-country panel data in a dynamic GMM model, López-Villavicencio and Mignon (2011) find that there exist a threshold level of inflation below which higher inflation results in higher economic growth.

There is also a variety of studies that analyze the effects of inflation in models of endogenous growth. In this vein, Jones and Manuelli (1995) review several growth models, including the AK model and the model with human capital accumulation.

[1] A non exhaustive list includes Barro (2001), Bruno and Easterly (1998), Chari et al. (1996), Gillman and Kejak (2005), Gylfason and Herbertsson (2001), Kormendi and Meguire (1985), McCandless and Weber (1995), and Walsh (2010, Chapter 1).

A role of money is introduced with the help of a cash-in-advance (CIA) constraint on consumption. In their models, they find relative modest effects of inflation on economic growth. Inflation distorts the leisure-consumption choice of the households so that labor decreases. The same mechanism is at work in Gomme (1993) and Wu and Zhang (1998) where higher inflation results in lower employment and, hence, less economic growth. In the two-sector monetary growth model of Maußner (2004), the effect of inflation on the growth rate depends on the value of the intertemporal rate of substitution. A value smaller than one implies a negative relation between the growth rate of money supply and the growth rate of per-capita income. Gillman and Kejak (2005) study the sensitivity of two-sector endogenous growth models with respect to the specification of the CIA constraint. They find a robust negative effect of inflation on growth.

In our analysis, we study the sensitivity of this result with respect to the assumption of labor market frictions in three endogenous growth models: 1) the AK model, 2) the Lucas (1990) supply-side model, and 3) the model with human capital accumulation as in Jones and Manuelli (1995).[2] All models are calibrated for the US economy and have the property that they are able to replicate the above finding that inflation reduces growth when we use the standard CIA constraint on consumption c and assume Walrasian labor markets. The mechanism is well-known: In the case of the standard CIA constraint, higher inflation introduces an inflation tax on the labor supply and employment declines. As a consequence, higher inflation has a negative effect on economic growth.

To illustrate this effect, consider the market clearing condition in the AK model with Walrasian labor markets:

$$w = [1 + (r + \pi)]MRS,$$

where w, r, π, and MRS denote the real wage, the real interest rate, the rate of

[2]Our choice of growth models is motivated by the fact that they represent the standard models considered in similar inflation-growth studies with Walrasian labor markets as in Gomme (1993), Jones and Manuelli (1995), or Gillman and Kejak (2005).

inflation, and the marginal rate of substitution between hours and consumption, respectively. The nominal interest rate factor $1 + r + \pi$ drives a wedge between the real wage and the marginal rate of substitution (the inflation tax). Therefore, a higher rate of inflation reduces employment, and, in the end, economic growth.

As an alternative specification of money demand, we will also introduce a cash-credit good as in the Dotsey and Ireland (1996) model so that the labor market clearing condition becomes

$$w = [1 + (r + \pi)(1 - \zeta)]MRS.$$

ζ is the endogenously determined number of goods purchased on credit. With increasing inflation the costs of holding real money balances m increase and the household accumulates less money. The CIA constraint $m \geq (1 - \zeta)c$ becomes more severe. If ζ were fixed, the household would have to reduce consumption. Instead, he can acquire more goods on credit and can, thus, relax the CIA constraint. Depending on the relative strength of the impact of π on the wedge and the ensuing increase in the share of credit goods, employment may either decline or increase. The size of the response of ζ to an increase of the inflation rate depends on the semi-interest elasticity of income velocity $\epsilon_{v,r+\pi}$.

If we assume search unemployment rather than Walrasian labor markets, we alter the transmission channel from inflation to growth. In search models wages result from a Nash bargaining process and depend on the reservation wage of the workers. With increasing inflation households decrease their cash balances and, thus, consumption. As a consequence, the marginal rate of substitution between consumption and working hours decreases, which in turn reduces the reservation wage and, hence, equilibrium wages. Firms find it more profitable to post vacancies. In addition, the households reduce their search effort in order to increase leisure time. Both effects increase the number of vacancies relative to the number of searching agents and, thus, the probability of finding a job, so that employment increases. Whether this effect prevails and ultimately enhances economic growth depends on the specific engine of growth and the modeling of the CIA constraint.

The paper is structured as follows. In Section 2, we present the AK model with two different specifications for the cash-in-advance constraint. In Section 3, we study the balanced growth equilibrium. In Section 4, we analyze two models of human capital accumulation. In the first model, human capital increases with the time spent on learning as in Lucas (1990). In the second model, we follow Jones and Manuelli (1995) where the households use goods in order to increase human capital. Section 5 provides a short summary of our results how inflation affects growth and unemployment. In Section 6, we present a welfare analysis of inflation effects. Section 7 concludes. The results for the models with Walrasian labor markets, and the systems of the equilibrium equations for all models are described in more detail in the Appendix.

2 The model

The model introduces money demand and search unemployment in the endogenous growth model of Romer (1986). Money is incorporated by assuming that households finance part of their consumption by using cash. The other part of consumption is financed by credit that is either free of transactions costs or costly as in the economy of Dotsey and Ireland (1996). In order to study the effects of inflation on unemployment, we consider frictions in the labor market in the form of search unemployment and wage bargaining following Shi and Wen (1997, 1999). Unemployment results from time-consuming and costly matching of vacancies with agents who are searching for a job.

The economy consists of four sectors: households, production firms, financial intermediaries, and the monetary authority. The representative household maximizes his expected intertemporal utility subject to his budget constraint and a CIA constraint. Firms produce a consumption-investment good using capital and labor. In order to hire workers, firms post vacancies. Financial markets provide credit services, while the central bank supplies money.

2.1 Households

A single household $h \in [0,1]$ consists of different members who are either employed, searching for a job, or enjoying leisure. The members pool their income. Let n and s denote the fraction of employed and searching household members, respectively. An employed person looses his job with an exogenously given probability of θ. Searching households will find a job with probability ξ, so that the share of employed households (both for each $h \in [0,1]$ and for the unit mass of households) evolves according to

$$\dot{n} = \xi s - \theta n. \tag{2.1}$$

The share of household members that enjoy leisure is equal to $x = 1 - n - s$.

Households maximize lifetime utility

$$\int_0^\infty u\left(c, x\right) e^{-\rho t} dt, \tag{2.2}$$

where c and ρ denote consumption and the discount rate of the household, respectively.

Leisure x increases utility, or, equally, searching and working, $s + n = 1 - x$, causes disutility to the household. Following Shi and Wen (1999), we parameterize the current-period utility function as follows:

$$u(c, x) = \ln c - \beta \frac{(1-x)^\eta}{\eta}. \tag{2.3}$$

Consumption financing. Consumers can purchase consumption with either cash or credit as in Schreft (1992), Gillman (1993), or Dotsey and Ireland (1996). The specific modeling of the transactions technology follows Heer et al. (2011). The consumption goods are indexed by $i \in [0,1]$, and the consumption aggregator is given by $c = \inf_i\{c(i)\}$. Therefore, the individuals will consume the same amount of all goods.

In the first specification of the monetary economy, the fraction $\zeta \in [0,1)$ of consumption goods is financed on credit, while the fraction $1 - \zeta$ of goods is financed with cash:

$$c(1-\zeta) \leq \frac{M}{P}, \tag{2.4}$$

where M and P denote nominal money and the price level, respectively. The inflation rate is defined by $\pi \equiv \frac{\dot{P}}{P}$.[3] In this case, the credit costs are zero and the fraction ζ is given exogenously.

In the second specification that follows Heer et al. (2011), ζ is determined within the model: In order to buy good i on credit, the household must pay the fee $q(i)$ to a financial intermediary who certifies the seller of the good that the household is worthy of credit. The financial intermediary requires $w\kappa(i)$ units of output to produce this service, where w denotes the real wage. With free market entry in this sector $q(i) = w\kappa(i)$. The function $\kappa(i)$ is strictly increasing in i, and satisfies $\lim_{i\to 1}\kappa(i) = \infty$. According to the latter assumption, some goods will be purchased with cash, and the demand for money is well defined. In particular, we parameterize the transaction technology as:

$$\kappa(i) = \kappa_0 \left(\frac{i}{1-i}\right)^{\chi}. \tag{2.5}$$

Budget constraint. In addition to the CIA constraint (2.4) and the employment dynamics (2.1), households face a budget constraint. They receive income from capital k, labor n, profits Ω, and real lump-sum transfers τ from the monetary authority. Real assets a consist of capital k and real money balances $m \equiv M/P$ and accumulate according to:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wn + rk + \Omega + \tau - c - \pi m & \text{case 1} \\ wn + rk + \Omega + \tau - c - \pi m - \int_0^{\zeta} q(i)\, di & \text{case 2} \end{cases} \tag{2.6}$$

The household maximizes (2.2) subject to (2.1), (2.4)-(2.6), and given initial values of capital, employment, and real money balances, k_0, n_0, and m_0, respectively.

2.2 The monetary authority

The economy-wide nominal money supply $M = Pm$ grows at the rate μ:

$$\frac{\dot{M}}{M} = \mu. \tag{2.7}$$

[3]Since the analysis only considers the situation where π is larger than the negative real interest rate r, $\pi \geq 0 > -r$, equation (2.4) will always hold as an equality at an optimum.

The seignorage obtained from money creation is paid to the households as a lump-sum transfer implying:

$$\tau = \mu m. \tag{2.8}$$

2.3 Firms

Firms are identical and of measure one. They use labor n and capital k in order to produce the consumption-investment good y with the technology $f(k, \bar{k}, n)$. The externality in aggregate capital accumulation $\bar{k}$ (which equals k in equilibrium) results in constant returns to capital as in Romer (1986):

$$y = Ak^{\alpha} n^{1-\alpha} \bar{k}^{1-\alpha}. \tag{2.9}$$

Workers separate from the representative firm with probability θ. To attract new workers the firm posts vacancies v at cost ϕwv. The probability that a vacancy is filled is ϑ. Therefore, employment at the firm level (and, in equilibrium, in the entire economy) evolves according to

$$\dot{n} = \vartheta v - \theta n \tag{2.10}$$

and profits are given by

$$\Omega = y - wn - (r + \delta)k - \phi wv. \tag{2.11}$$

The firm maximizes discounted profits

$$\int_0^{\infty} \Omega e^{-\int_0^t r(\xi) d\xi} dt, \tag{2.12}$$

subject to (2.10), (2.11), the specification of the production function in (2.9), and a given initial employment n_0.

2.4 Matching and bargaining

At the aggregate level, the mass of successfully filled vacancies M is determined by

$$M = v^{\gamma} s^{1-\gamma}, \tag{2.13}$$

where v and s denote the mass of vacancies and the share of searching households. Since households cannot search while being employed,[4] the unemployment rate u is given by

$$u = \frac{s}{n+s}. \tag{2.14}$$

The probability for a household of finding a job ξ equals

$$\xi = \frac{M}{s} = (v/s)^{\gamma} \tag{2.15}$$

and the probability that an open vacancy is filled follows from

$$\vartheta = \frac{M}{v} = (v/s)^{\gamma-1}. \tag{2.16}$$

Wages result from decentralized Nash bargaining between the firm and the marginal worker. In particular, the wage w maximizes

$$\max_{w} \left[MPL - w\right]^{1-\lambda} \left[w - MRS\right]^{\lambda} \tag{2.17}$$

where MPL denotes the marginal product of labor and MRS the marginal rate of substitution between working hours and consumption. The Nash bargaining solution is given by:

$$w = \lambda(1-\alpha)Ak^{\alpha}n^{-\alpha}\bar{k}^{1-\alpha} + (1-\lambda)\beta(n+s)^{\eta-1}c. \tag{2.18}$$

2.5 Balanced growth path

The stationary equilibrium or the balanced growth path of the model has the following properties:

1. The stock of capital k, real money balances m, output y, consumption c, and the real wage grow w at the same constant rate g.

[4] We refrain from introducing on-the-job search. The motive to search on the job originates from wage dispersion, and, hence heterogeneity among the firms. To deal with this additional complexity would require an entirely different set up which we leave as an interesting subject for future research.

2. The rental rate of capital r, the share of goods purchased with credit ζ, the inflation rate π, the unemployment rate u, and, thus, employment n and the fraction of unemployed households s, and the mass of vacancies v are constant.

The growth rate and the stationary values of the variables are determined by the system of equations (2.19), in which the tilde ˜ refers to variables scaled by k:

$$g = An^{1-\alpha} - \delta - \tilde{c} - \tilde{w}\left(\phi v + \kappa_0 \int_0^\zeta \left(\frac{i}{1-i}\right)^\chi\right), \tag{2.19a}$$

$$g = r - \rho, \tag{2.19b}$$

$$g = \mu - \pi, \tag{2.19c}$$

$$r = \alpha A n^{1-\alpha} - \delta, \tag{2.19d}$$

$$\tilde{w} = \lambda \alpha n^{-\alpha} + (1-\lambda)\beta(n+s)^{\eta-1}\tilde{c}, \tag{2.19e}$$

$$(v/s)^\gamma = \theta(n/s), \tag{2.19f}$$

$$\tilde{c}(r+\pi) = \tilde{w}\kappa_0\left(\frac{\zeta}{1-\zeta}\right)^\chi, \tag{2.19g}$$

$$g = r + \theta - \frac{(v/s)^{\gamma-1}}{\phi}\left(\frac{(1-\alpha)An^{-\alpha}}{\tilde{w}} - 1\right), \tag{2.19h}$$

$$\tilde{w} = [1 + (r+\pi)(1-\zeta)]\beta(n+s)^{\eta-1}\tilde{c}\left((\theta+\rho)(v/s)^{-\gamma} + 1\right). \tag{2.19i}$$

Equation (2.19a) is the economy's resource constraint which derives from the household's budget constraint (2.6), the definition of profits (2.11), and the zero profit condition for financial intermediation. Equation (2.19b) derives from the household's Euler equation for capital accumulation. Equation (2.19c) follows from the CIA constraint (2.4). Equation (2.19d) is the firm's first-order condition for the demand of capital. Equation (2.19e) solves the Nash bargaining problem (see (2.18)). Equation (2.19f) combines (2.1) for $\dot{n} = 0$ and (2.15). Equation (2.19g) is the first-order condition for the optimal share of credit goods. Equation (2.19h) combines the firm's first-order condition for vacancies and the growth rate of the firm's shadow price of employment. Finally, equation (2.19i) is implied by the household's shadow price of employment and the first-order condition for the time spent searching.[5]

[5] See the Appendix for details.

It is obvious from equation (2.19d) that in a model with Walrasian labor markets and exogenous labor supply n the growth rate only depends on the four parameters A, α, n, and δ. In this case, money is superneutral. Thus, the effect of money on employment is decisive in understanding the inflation-growth nexus.

2.6 Calibration

The effects of a change in the inflation rate (as resulting from a change in the growth rate of money supply) cannot be studied analytically but only numerically.[6] For this reason, the model is calibrated in order to match characteristics of the US economy. The unit time length corresponds to one quarter. In case 1 (case 2), we have to find the values of 12 (13) parameters. For a subset of 8 parameters, $\{\alpha, \eta, \rho, \delta, \mu, \theta, \gamma, \lambda\}$, we use observations to calibrate them individually. Most of these values, namely α, δ, η, and ρ, are taken from the literature such as Kydland and Prescott (1982). Their empirical estimates are based on time series evidence covering subsamples of the post world war II period. For the remaining parameters, i.e. either $\{\beta, A, \zeta, \phi\}$ in case 1 or $\{\beta, A, \kappa_0, \chi, \phi\}$ in case 2, we calibrate them simultaneously such that both a set of empirical observations and the equilibrium conditions (2.19) hold.

We set the production elasticity of private capital equal to $\alpha = 0.36$ and the quarterly rate of capital depreciation equal to $\delta = 0.025$. The household discounts future utilities at the rate $\rho = 0.01$. $\eta = 3.5$ implies a labor supply elasticity of $\epsilon_{n,w} = 1/(\eta - 1) = 0.4$, and the value of μ follows from our inflation target, which is either 0 or 10 percent p.a. The parameters $\{\theta, \gamma, \lambda\}$ are set as in Shi and Wen (1999).

The remaining parameters β, A, ϕ, and (in case 1) ζ are set so that

i. the labor force participation rate $n + s$ equals 68%,[7]

ii. the unemployment rate is equal to 6%,

[6] The GAUSS computer programs are available from the authors upon request.

[7] Both, the value for u and for $n + s$ are taken from Shi and Wen (1999).

iii. the annual growth rate of output $(1+g)^4 - 1$ (where g is the quarterly rate) is equal to 2%,

iv. the share of cash goods in total consumption is equal to 82 percent.[8]

In case 2, we use yet another observation in addition to (iv) to simultaneously determine κ_0 and χ: the semi-interest elasticity of the income velocity of money $\epsilon_{v,r+\pi}$ implied by the model must equal 5.95 as estimated by Dotsey and Ireland (1996) for the US economy during 1959-1991. The values of the exogenously given parameters are summarized in Table 2.1.

Table 2.1
Calibration of the benchmark model

Preferences	$n+s$=0.68	η=3.5	ρ=0.01
Production	α=0.36	δ=0.025	
CIA	ζ=0.18	$\epsilon_{v,r+\pi}$=5.95	
Labor market	θ=0.05	γ=0.6	λ=0.6
	u=0.06		
Output growth	$(1+g)^4-1$=0.02		
Money supply	$(1+\pi)^4-1$=0		

3 Inflation and economic growth

In this section, we study the effects of inflation on economic growth in the model of Section 2. We consider annual inflation rates between 0% and 15% p.a.[9] In this range the rate of growth increases with inflation. This result holds for both specifications of the CIA constraint. In the version of our model with Walrasian labor markets (which we derive in the Appendix), the inflation-growth relation depends on the kind of the

[8] This value for the US economy is found by Avery et al. (1987).

[9] The choice of this range is motivated by the fact that the post world war II regimes of monetary policy fall within this interval. For example, the average inflation rates during the Volker (1979-1987) and Greenspan eras (1988-2003) amounted to 6.43% and 3.06%, respectively.

CIA constraint: if the share of credit goods ζ is given exogenously, inflation depresses growth whereas it is growth enhancing if ζ is determined within the model.

In the second part of this section, we study the sensitivity of our result for the economy in case 2 with respect to the magnitude of the semi-interest elasticity of income velocity.[10] We show that the inflation-growth trade-off does not depend on this parameter in the AK model. For the models with human capital accumulation that we consider in the next section, however, this parameter will be crucial.

Moreover, we show that unemployment declines initially with rising inflation irrespective of the form of the cash-in-advance constraint.

3.1 Specification of the cash-in-advance constraint

Figure 3.1 displays the relation between the annual rate of inflation and various endogenous variables along a balanced growth path. In each of the six panels the annual inflation rate on the abscissa increases from 0% to 15% p.a. The respective left ordinates correspond to lines in black while the right ones refer to the blue lines. The figure depicts three cases:

Case 1: refers to the model with search frictions and an exogenously fixed share of credit goods,

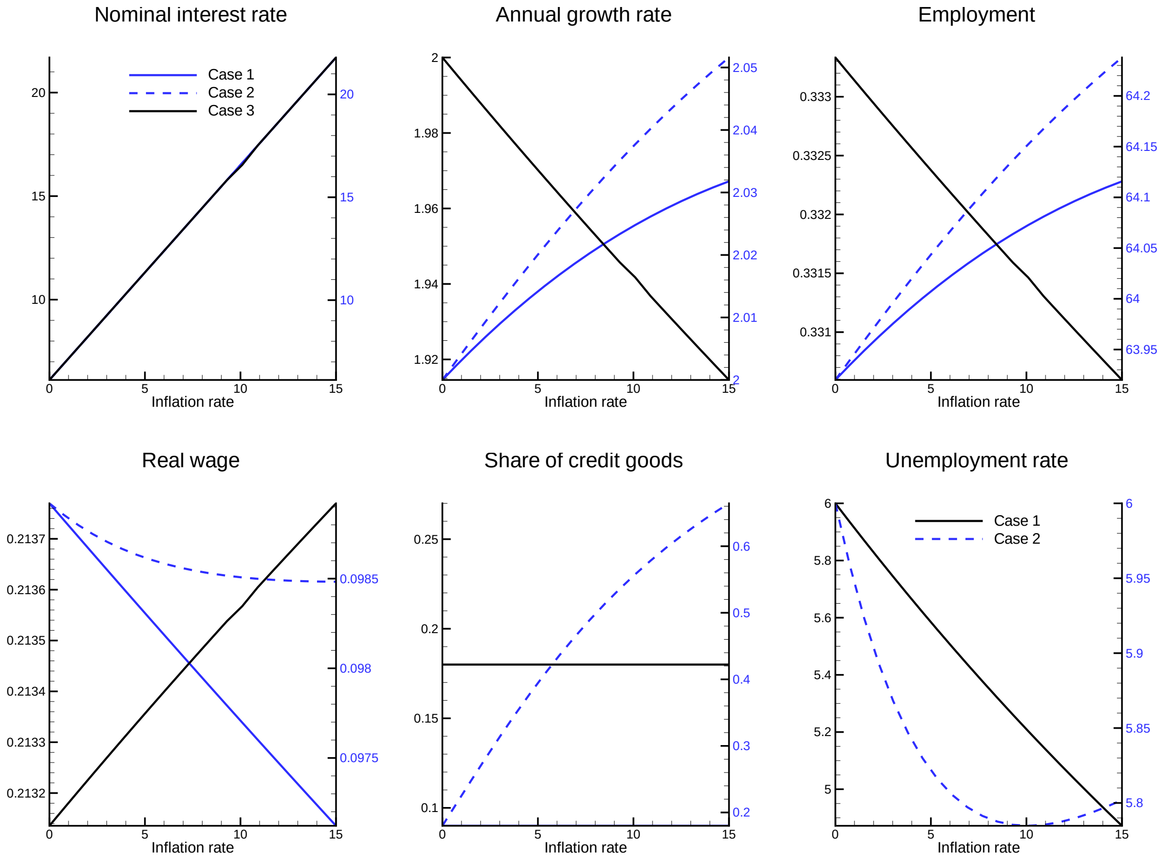
Case 2: refers to the model with search frictions and an endogenously determined share of credit goods,

Case 3: refers to the model with a Walrasian labor market and an exogenously fixed share of credit goods.

The upper right panel of Figure 3.1 displays the behavior of the nominal interest rate $r + \pi$. In cases 1 and 2 it increases from 6.12% to 21.79% p.a. In case 3 it reaches

[10] In the Appendix, we also study the sensitivity of the growth effects on the separation rate θ. However, the growth rate is not affected by its magnitude.

Figure 3.1: Inflation and growth in the AK model



21.69% for an inflation rate of 15%. The nominal interest rate distorts the household's consumption-leisure choice. With Walrasian labor markets this choice is determined by

$$w = [1 + (r + \pi)(1 - \zeta)]c\beta n^{\eta - 1}.$$

Thus, for a given real wage w and a fixed share of credit goods, a higher nominal interest rate requires less consumption and less working hours. As shown by the black line in the rightmost upper panel, hours decrease with inflation. It is then obvious from equations (2.19d) and (2.19b) (which also hold in the case of Walrasian labor markets) that the rate of growth must decline.

With search frictions, the previous equation is replaced by equation (2.19i) and two additional effects arise. First, the household may increase leisure by lowering his search effort. Second, the probability of finding a job ξ is a decreasing function of labor market tightness s/v. Inflation – by increasing the costs of money holdings – reduces consumption and thus the household's reservation wage (see the lower leftmost panel in Figure 3.1). Firms find it more profitable to post vacancies. This effect is reinforced by the lower search effort of households so that ξ increases (not shown in the Figure) raising employment. Obviously, for our calibration, this effect dominates the effect that results from the household's desire to increase leisure due to increased nominal interest rates. Therefore, employment and growth increase with inflation.

A third effect appears in the case of credit costs. It reinforces the positive effect on employment (compare case 1 and case 2 in the rightmost upper panel of Figure 3.1) but dampens the effect on unemployment. In his desire to reduce the costs of money holdings, the household buys more goods on credit so that the wedge $[1+(r+\pi)(1-\zeta)]$ in equation (2.19i) declines. As a consequence, the household is not forced to reduce consumption by as much as in the case of a fixed share of credit goods. For this reason, wages decline less, and beyond a certain threshold for inflation even start to increase. This also explains the u-shaped behavior of the rate of unemployment: Higher wages deter firms from posting vacancies while they encourage households to search for jobs. The labor market becomes tighter decreasing the probability of finding a job.

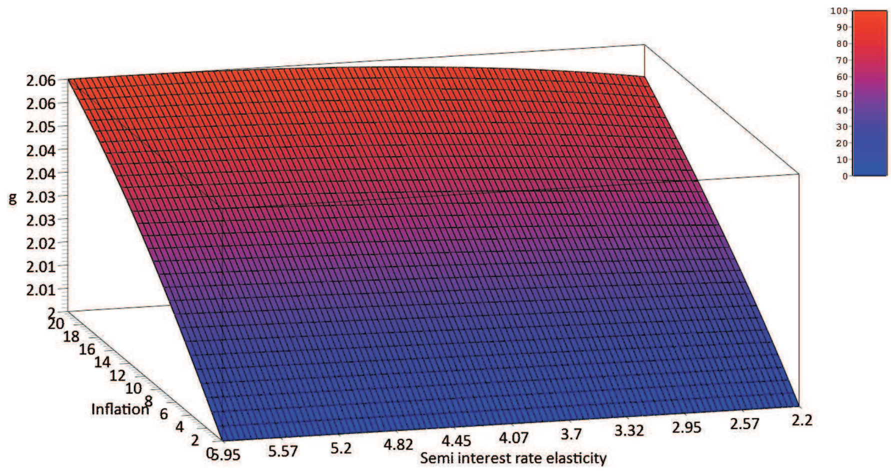
3.2 Sensitivity with respect to the semi-interest elasticity

In Figure 3.2, we study the sensitivity of the growth effects of inflation for the benchmark model of Section 2 (case 2) with respect to the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. For the value estimated by Dotsey and Ireland (1996), $\epsilon_{v,r+\pi} = 5.95$, the growth-enhancing effect is more pronounced than for lower values, $\epsilon_{v,r+\pi} = 2.20$.[11] If the income velocity is very sensitive to the interest rate (higher values of $\epsilon_{v,r+\pi}$), households reduce their cash holdings more markedly with rising inflation, or, equally, ζ is more sensitive with respect to inflation. As a consequence, the inflation costs are

[11] Dotsey and Ireland (1996) estimate $\epsilon_{v,r+\pi}$ from a simple regression of the log of velocity on the nominal interest rate using annual US data from 1959-1991. A much smaller value of $\epsilon_{v,r+\pi} = 0.1$ follows from Ball (2001), p. 40, who employs more sophisticated methods and who uses data from 1946-1996. Thus, $\epsilon_{v,r+\pi} = 2.2$ lies well in the range of empirically plausibles estimates.

lower and growth is higher for larger $\epsilon_{v,r+\pi}$. For the present calibration, the effect, however, is very small as can be seen by comparing the inflation-growth trade-off for the values $\epsilon_{v,r+\pi} = 5.95$ and $\epsilon_{v,r+\pi} = 2.20$. The growth rate amounts to 2.063% and 2.054%, respectively, if inflation increases to 15%.

Figure 3.2: Growth and Inflation in the AK Model: Sensitivity with respect to $\epsilon_{v,r+\pi}$



4 Endogenous growth and human capital

In the following, we examine the sensitivity of our results with respect to the specification of the engine of growth. In Lucas (1990) hours spent on learning increase the stock of human capital, which is used along with raw labor and physical capital to produce consumption and investment goods. Jones and Manuelli (1995) assume that goods rather than time are used to accumulate human capital. In this section, we only describe the essential new features of these endogenous growth models in comparison to the benchmark AK model in Section 2. A full description of the models and their equilibrium conditions is delegated to the Appendix.

4.1 The Lucas (1990) supply-side model

To keep the description of the model as brief as possible, we just present the maximization problems of the household and the firm sector, respectively. Everything else remains unchanged.[12]

Households. The representative household can allocate his time endowment 1 to work n, searching s, learning l, or leisure x:

$$1 = n + s + l + x. \tag{4.1}$$

Human capital of the individual h is determined by the time l he allocates to learning:

$$\dot{h} = Dhl^{\nu}. \tag{4.2}$$

The individual's labor income is given by the product of the wage, w, and his effective labor, hn. In case 2, costs per unit of credit service are proportional to the costs of labor given by $w\bar{h}$, where $\bar{h}$ is the stock of human capital of the average worker.[13] The household's budget constraint, thus, is:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wnh + rk + \Omega + \tau - c - \pi m & \text{case 1,} \\ wnh + rk + \Omega + \tau - c - \pi m - \int_0^{\zeta} q(i)\, di & \text{case 2.} \end{cases} \tag{4.3}$$

The household maximizes (2.2) subject to (2.4), (2.5), (4.2), and (4.3). The first-order and equilibrium conditions of the model are derived in the Appendix.

Firms. Output is produced using capital k and effective labor, hn, according to:

$$y = Ak^{\alpha}(hn)^{1-\alpha}. \tag{4.4}$$

In the Lucas model, the vacancy costs must rise with the human capital of the workers in the financial sector implying the profits

$$\Omega = Ak^{\alpha}(nh)^{1-\alpha} - (r + \delta)k - wnh - \phi whv. \tag{4.5}$$

[12] For a more detailed presentation of the model, see Lucas (1990) and Grüner and Heer (2000).

[13] Note that in equilibrium $\bar{h} = h$.

Firms maximize

$$\int_0^\infty \Omega e^{-\bar{r}(t)} dt$$

subject to (2.10) and (4.5). Otherwise, the production sector is identical to that in the model of Section 2.

Calibration. As compared to the AK model the Lucas model has two additional parameters, D and ν. We employ the value of $\nu = 0.8$ from Lucas (1990). D is calibrated so that the steady-state annual rate of growth equals 2% for $\pi = 0\%$. In addition, we set $A = 1$.[14]

4.2 The model of Jones and Manuelli (1995)

The model. As a second variation of the endogenous growth model with human capital, we assume that human capital accumulation requires goods rather than time:

$$\dot{h} = i_h - \delta_h h, \tag{4.6}$$

where i_h and δ_h denote investment in human capital and the rate of human capital depreciation, respectively. As a consequence, the household's budget constraint is modified to:

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wnh + rk + \Omega + \tau - c - \pi m - i_h & \text{case 1,} \\ wnh + rk + \Omega + \tau - c - \pi m - i_h - \int_0^\zeta q(i)\, di & \text{case 2.} \end{cases} \tag{4.7}$$

Households maximize (2.2) (with the current-period utility function specified by (2.3)) subject to (2.4), (4.6) and (4.7).

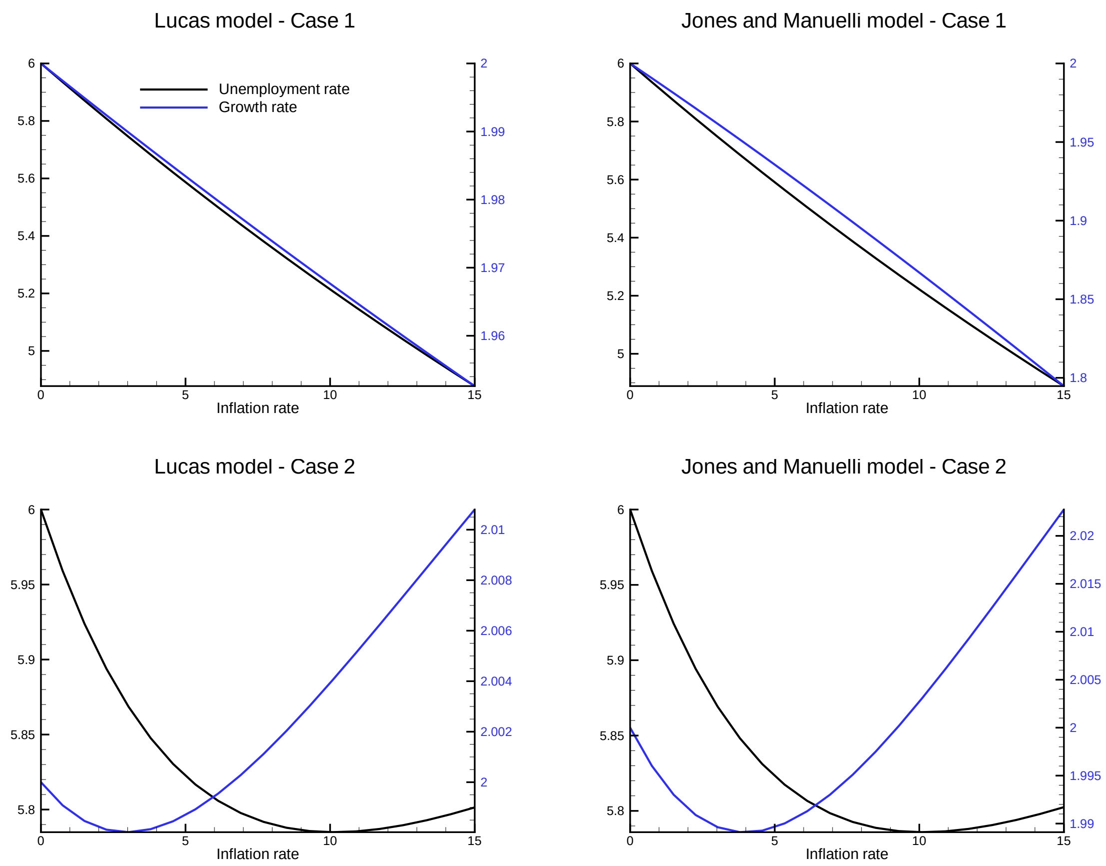
The equilibrium conditions of the firm are the same as in the Lucas model in Section 4.1. The stationary equilibrium conditions are described in more detail in the Appendix. We follow Jones and Manuelli (1995) and set $\delta_h = \delta$. The remaining parameters are either set to the values presented in Table **??** or obtained in the same way as explained in Section 2.

[14] The calibration exercise is a bit more involved for the Lucas model than for the AK model and is described in more detail in the Appendix.

4.3 Results

Effects on unemployment. Figure 4.1 displays the relation between the rate of inflation, the unemployment rate, and the rate of per-capita growth obtained from simulations of our two models for cases 1 and 2 (with $\epsilon_{v,r+\pi} = 5.95$). To distinguish the different models we use the shorthands L and JM for the models of Sections 4.1 and 4.2, respectively.

Figure 4.1: Inflation, unemployment, and growth in models with labor market frictions



In both models, the equilibrium unemployment rate declines with inflation in case 1 and is a u-shaped function of the rate of inflation in case 2. As in the AK model, these results are driven by two effects. First, with increasing inflation households decrease their cash balances and, thus, consumption. As a consequence, the marginal rate of substitution between consumption and working hours decreases, which in turn reduces the reservation wage. Firms find it more profitable to post vacancies. In addition, the households reduce their search effort in order to increase leisure time. Both effects decrease labor market tightness s/v so that the probability of finding a job, and, in turn, employment increase. The net effect of less searching agents and a higher number of vacancies on employment is negative in the case of the JM model with constant ζ (case 1), and positive in all other cases. Second, in case 2, the households increase their share of credit goods, which mitigates the effect of inflation on consumption, and, thus, on the real wage. If inflation exceeds a threshold of about 10%, this effect reverses the first one.

Effects on the rate of growth. Different from the AK model, the effect of employment does not necessarily mimic the effect on the rate of growth. In the Lucas model, the fraction of the time endowment devoted to human capital accumulation l determines the rate of growth (see equation (4.2)). For low rates of inflation, the household demands more leisure, and since the marginal benefit of employment increases, the household substitutes market work for time spent to increase human capital. For higher rates of inflation, the increasing share of credit goods decreases the demand for leisure, and the household devotes more time to increase his human capital. The threshold level of inflation for this effect to dominate the overall effect is about 3% p.a.

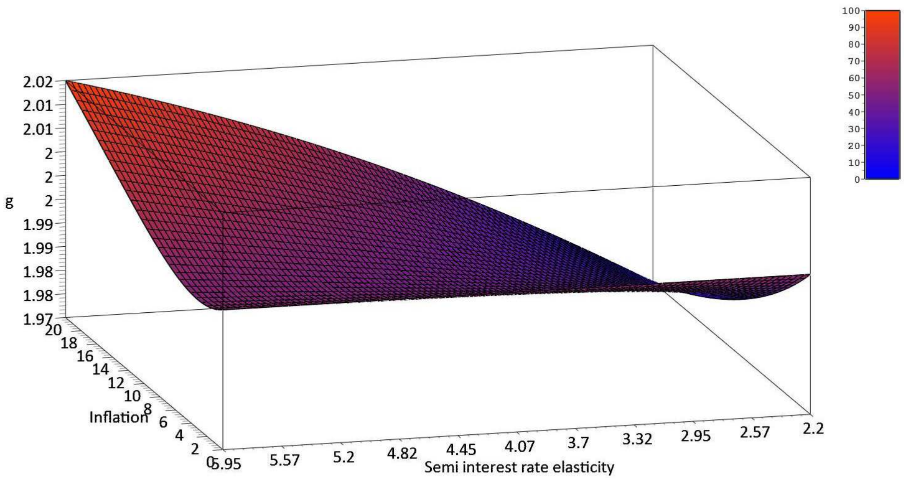
In the Jones and Manuelli model the trade-off between investment in physical, human, and monetary capital determines the effect on the rate of growth. With increasing inflation, the household substitutes physical and human capital for monetary balances. The household invests in human capital until its return $wn - \delta_h$ equals the return on physical capital r.[15] In case 2, there is again a threshold level of inflation of about 4% p.a. at which the decline of wn comes to a halt. Beyond this threshold, the portfolio reallocation increases the rate of growth.

Sensitivity. Figures 4.2 and 4.3 show that the u-shaped effects of inflation on the rate of growth are sensitive with respect to the semi-interest elasticity of the income

[15]Note, in the Jones and Manuelli model, physical capital, human capital, and consumption are perfect substitutes so that there are no relative price effects.

velocity of money $\epsilon_{v,r+\pi}$. Different from the AK model, there are threshold values of $\epsilon_{v,r+\pi}$ below which inflation reduces the rate of economic growth over the entire range of inflation rates considered. In the Lucas model the threshold is at $\epsilon_{v,r+\pi} = 2.2$ while it occurs at $\epsilon_{v,r+\pi} = 3.3$ in the Jones and Manuelli model. The qualitatively different behavior of the growth rate below these thresholds in both models of human capital accumulation as compared to the AK model demonstrates the central role played by the engine of growth rather than the modeling of money demand.

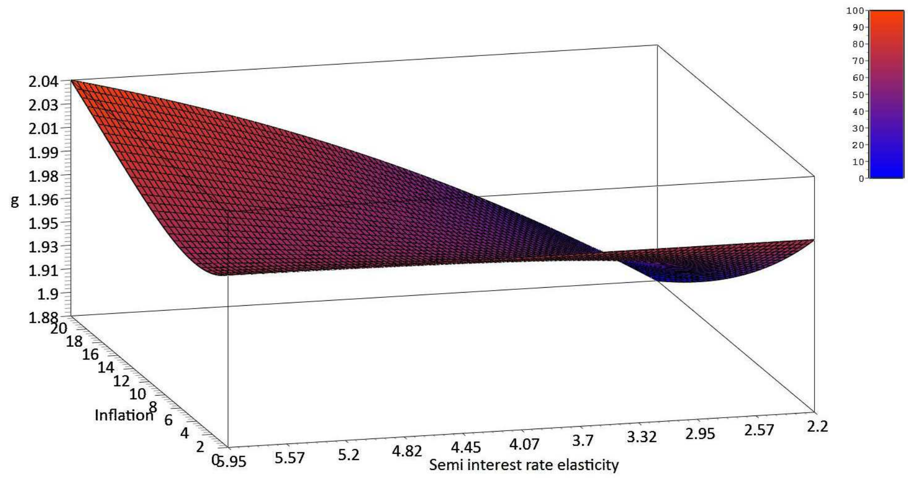
Figure 4.2: Growth and inflation in the Lucas model: sensitivity with respect to $\epsilon_{v,r+\pi}$



5 Summary of results

Our results of the inflation effects on growth and unemployment are summarized in Tables 5.1 and 5.2. The entries display the economic growth rate and the rate of unemployment, respectively, for 10% annual inflation. The benchmark values for zero inflation are 2% and 6%, respectively. In the first entry column, the numbers refer to the models with the standard CIA constraint. Depending on the endogenous growth mechanism, economic growth may increase (in the case of the AK model) or decrease (in the case of the Lucas and Jones/Manuelli models). Notice, however, that the magnitude of the quantitative effect on growth is small. For example, in the AK

Figure 4.3: Growth and inflation in the Jones and Manuelli model: sensitivity with respect to $\epsilon_{v,r+\pi}$



model, economic growth only increases from 2.00% to 2.025% if inflation rises from 0% to 10%. The effects on the rate of unemployment are much stronger with a minimum impact of 0.21 and a maximum of 0.79 percentage points.

The impact of inflation on economic growth and unemployment depends on the specification of the alternative payment means.[16] In the case of the CIA constraint in Dotsey and Ireland (1996), consumers purchase consumption goods with either cash or credit where the latter is subject to transaction costs in the financial market. If inflation increases, households buy a smaller number of goods with cash. In the AK model this mechanism strengthens the positive effect of inflation on growth and reverses the negative growth effects of inflation in the Lucas and Jones and Manuelli model, if the semi-interest elasticity of income velocity is not too small. This can be seen by comparing the first with the second and third entry columns of Table 5.1.

[16]For monetary growth models with Walrasian labor markets, this finding has also been pointed out by Gillman and Kejak (2005).

Table 5.1
Effects of inflation on per-capita income growth

Model	CIA	DI	
		$\epsilon_{v,r+\pi}$	
		5.95	2.20
AK	2.025%	2.037%	2.031%
Lucas (1990)	1.968%	2.004%	1.983%
Jones and Manuelli (1995)	1.867%	2.003%	1.928%

Notes: The columns display the growth rate after an increase of the inflation rate π from 0% to 10% in the three prototype endogenous growth models for different specifications of the cash-in-advance constraint. The first entry column displays the results for the standard cash-in-advance (CIA) constraint. In the second and third entry column under the caption DI, the results are derived in the cash-credit economy for two different values of the semi-interest elasticity of income velocity $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).

6 Welfare analysis

In this section, we provide a welfare analysis of the inflation effects.[17] Based on Lucas (1987), we express the welfare change associated with an increase of inflation from 0% to 10% by the change in total consumption (as percentage of output), Δ, that is necessary to make the individual in the old equilibrium indifferent between these two steady states. In addition, following Gomme (1993), we decompose the welfare effects in the three factors affecting steady-state life-time utility: 1) consumption, 2) leisure, and 3) the growth rate. Our welfare results are summarized in Table 6.1.

To illustrate the decomposition of the welfare effects, let $\{\tilde{c}, \tilde{y}, x, g\}$ ($\{\tilde{c}^a, \tilde{y}^a, x^a, g^a\}$) denote the steady-state values of consumption, output, leisure, and the growth rate in the steady state with zero annual inflation (10% inflation), where variables with a tilde are expressed as the respective levels relative to capital, e.g. consumption per capita

[17] We would like to thank an anonymous referee for suggesting this analysis to us.

Table 5.2
Effects of inflation on the rate of unemployment

Model	CIA	DI	
		$\epsilon_{v,\pi}$	
		5.95	2.20
AK	5.21%	5.79%	5.45%
Lucas (1990)	5.21%	5.79%	5.45%
Jones and Manuelli (1995)	5.22%	5.79%	5.46%

Notes: The columns display the rate of unemployment after an increase of the inflation rate π from 0% to 10% in the three prototype endogenous growth models for different specifications of the cash-in-advance constraint. The first entry column displays the results for the standard cash-in-advance (CIA) constraint. In the second and third entry column under the caption DI, the results are derived in the cash-credit economy for two different values of the semi-interest elasticity of income velocity $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).

$c = \tilde{c} \cdot k = c \cdot k_0 e^{gt}$ grows at rate g. Given the specification of life-time utility in (2.2), the welfare effect Δ can be computed from the following equation:[18]

$$\begin{aligned} W(\tilde{c} + \Delta\tilde{y}, x, g) &= \int_0^\infty \ln(c + \Delta y) - \beta \frac{(1-x)^\eta}{\eta} e^{-\rho t}\, dt \\ &= \frac{\ln(\tilde{c} + \Delta\tilde{y})}{\rho} + \frac{\ln k_0}{\rho} + \frac{g}{\rho} + \beta \frac{(1-x)^\eta}{\rho\eta} \\ &\stackrel{!}{=} W(\tilde{c}^a, \tilde{y}^a, x^a, g^a) = \frac{\ln \tilde{c}^a}{\rho} + \frac{\ln k_0}{\rho} + \frac{g^a}{\rho} + \beta \frac{(1-x^a)^\eta}{\rho\eta}. \end{aligned}$$

A Taylor-series approximation then implies

$$\Delta = \frac{\tilde{c}}{\tilde{y}} \left\{ [\ln \tilde{c}^a - \ln \tilde{c}] + \frac{\beta}{\eta} [(1-x)^\eta - (1-x^a)^\eta] + \frac{g^a - g}{\rho} \right\}.$$

The entries in Table 6.1 labeled 'Consumption', 'Leisure', and 'Growth rate' correspond to the three additive terms on the right-hand side of this equation.

The following results presented in Table 6.1 are noteworthy:

[18]We assume that, under both scenarios for the inflation rate, the economy starts with the same initial capital stock, k_0, at time 0.

Table 6.1
Effects of inflation on welfare

Model	Walras			Search		
	CIA	DI		CIA	DI	
		$\epsilon_{v,\pi}$			$\epsilon_{v,\pi}$	
		2.2	4.95		2.2	4.95
AK						
Total	−0.926	−0.732	−0.485	0.282	0.133	−0.113
Consumption	−0.230	−0.417	−0.656	−0.455	−0.535	−0.658
Leisure	0.352	0.161	−0.088	0.330	0.157	−0.073
Growth Rate	−1.048	−0.476	0.259	0.407	0.512	0.618
Lucas (1990)						
Total	0.032	−0.297	−0.722	−0.611	−0.693	−0.815
Consumption	−0.097	−0.357	−0.689	−0.656	−0.703	−0.775
Leisure	0.573	0.261	−0.143	0.580	0.288	−0.105
Growth Rate	−0.443	−0.201	0.109	−0.535	−0.278	0.065
Jones and Manuelli (1995)						
Total	0.000	−0.111	−0.243	−0.327	−0.295	−0.282
Consumption	0.000	−0.108	−0.244	−0.064	−0.145	−0.251
Leisure	0.033	0.190	−0.105	0.538	0.281	−0.047
Growth Rate	−0.034	−0.193	0.106	−0.801	−0.430	0.016

Notes: The column entries give the percentage of output that must be given or taken as additional consumption to the representative household in the zero inflation equilibrium so that he is indifferent between this equilibrium and the one with 10% inflation. The rows labeled consumption, leisure, and growth rate split the total welfare effect in those parts that can be attributed to the change in consumption, leisure, and the growth rate, respectively. CIA refers to the cash-in-advance model, DI to the Dotsey and Ireland (1996) model of money holdings.

1. In the Walras economies, the role of money demand, i.e. the specification of the CIA constraint, is crucial for the sign and the significance of the inflation effect. In both the two-sector models of Lucas (1990) and Jones and Manuelli (1995), the standard CIA constraint implies negligible welfare effects as in Gomme (1993).[19] However, if we consider the Dotsey-Ireland cash-credit economy with a high semi-interest-elasticity, $\epsilon_{v,\pi} = 4.95$, an inflation rate of 10% raises the welfare costs to 0.722% and 0.243% of total output in the two models, respectively.

2. The introduction of search unemployent in the two-sector endogenous growth models increases the magnitude of the welfare effects irrespective of the motive for money demand. For example, in the Lucas model with the cash-credit good and $\epsilon_{v,\pi} = 4.95$, the welfare loss of total consumption due to the higher inflation rate amounts to 0.815% of output rather than 0.722% in the case of Walrasian labor markets.

3. In comparison with the Walras model, we find that the effect of leisure remains significant when we introduce search unemployment. Therefore, we also find the same result as Gomme (1993) that "these results point to the importance of leisure in the calculation of welfare costs."

Alternative labor market specifications. To study the robustness of our welfare results, we consider two additional forms of matching mechanisms. In particular, we analyse 1) efficient bargaining and 2) wage posting.[20]In the first case, we consider bargaining over both the real wage and working hours as in Trigari (2006). In this case, both the wage and the optimal working hours are chosen to maximize the product of the weighted surpluses for the worker and the entrepreneur. In the second case, workers search for a job and decide to which job they should apply for. The probability of

[19]Gomme (1993) finds a welfare loss of 0.022% following an increase of the annual money growth rate from 0% to 10%.

[20]For a more detailed description of these models with the alternative specifications of the labor market, we refer the interested reader to the Appendix that is available from the authors upon request.

getting selected depends on the number of applicants. For this specification, we follow Shi and Wen (1999) who employ a matching game based on Peters (1991).

Table 6.2

Sensitivity of welfare effects with respect to wage setting

Model	Wage setting Wage bargaining	 Efficient bargaining	 Wage posting
AK			
Total	−0.113	−0.270	−0.943
Consumption	−0.658	−0.293	−0.791
Leisure	−0.073	−0.091	0.078
Growth rate	0.618	0.114	−0.230
Lucas (1990)			
Total	−0.815	−0.395	−0.760
Consumption	−0.775	−0.322	−0.690
Leisure	−0.105	−0.133	−0.132
Growth rate	0.065	0.060	0.062

In Table 6.2, we present the welfare effects resulting from these modifications in the AK-model and the Lucas (1990) model for the case of the DI constraint with $\epsilon_{v,\pi} = 4.95$. Notice that the magnitude and the direction of the welfare changes remain unaltered. In the case of the AK-model, the total welfare loss even increases and amounts to almost 1% in the case of wage posting. In the case of the Lucas (1990) model, the magnitude of the total effect is reduced, but still economically significant in both cases, 0.395% and 0.760% in the case of efficient bargaining and wage posting, respectively.

7 Conclusion

In our models of economic growth, we have emphasized the effects of search unemployment. As opposed to models with Walrasian labor markets, inflation has an additional transmission channel of inflation on economic growth in this case. Higher inflation affects the reservation wage of the households and increases labor market tightness. As a consequence, unemployment initially declines with inflation. Therefore, we find that inflation harms long-run economic growth to a smaller extent in monetary growth models with frictions than in those with Walrasian labor markets. While the magnitude of the growth rate effects are small, we find economically significant welfare effects of inflation. In the case of the two-sector models with human capital, a rise of inflation from 0% to 10% reduces steady-state life-time utility by 0.33% and 0.61% of total output in the two model variants, respectively.

In addition, we show that the specification of the cash-in-advance constraint matters. We compare monetary growth models with a standard cash-in-advance constraint to those with a costly-credit channel as in Dotsey and Ireland (1996). In the latter case, agents buy more goods with credit if inflation increases and economic growth is higher than in the case of the standard cash-in-advance constraint on consumption. Moreover, if households can choose the financing of consumption by means of cash and credit, the qualitative effect of inflation on economic growth becomes sensitive to the semi-interest elasticity of income elasticity for which the empirical evidence provides a broad range of values.

At this point, let us mention one word of caution. We have focused our analysis and also our discussion on models within a cash-in-advance context. Our results may be sensitive with respect to the introduction of other monetary frictions and, in particular, those that become more important for higher rates of inflation or during the process of economic development. For example, with high rates of inflation, transaction costs and employment in the financial sector as emphasized by Ireland (1994) are likely to result in lower employment and less growth. In a similar vein, inflation may increase transaction costs and therefore reduces growth in the models of De Gregorio (1993) and Jha et al. (2002). Furthermore, with higher inflation, the volatility of inflation is likely to increase. We intend to study the sensitivity of our results with respect to other monetary frictions in the future.

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Appendix

In Table A.1, the effects of an increase of the inflation rate from 0% to 10% are summarized for the three endogenous growth models AK, Lucas, and JM with Walrasian labor markets. In all three models with the standard CIA constraint on consumption, growth does not increase with higher inflation, while for the cash-credit economy, the direction of the inflation effect on growth depends on the magnitude of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$.

Table A.1
Summary: Effects of inflaton on per-capita income growth, Walrasian labor markets

	CIA	DI	
		$\epsilon_{v,r+\pi} = 5.95$	$\epsilon_{v,r+\pi} = 2.20$
AK model	1.942%	2.014%	1.974%
Lucas (1990)	1.975%	2.006%	1.989%
Jones and Manuelli (1995)	1.930%	2.017%	1.969%

Notes: The columns display the economic growth rate for an inflation rate π equal to 10% in the three endogenous growth models with Walrasian labor markets for different specifications of the cash-in-advance constraint. For the inflation rate of 0%, the economic growth rate is equal to 2.00% in all three cases AK, L, and JM. The first entry column displays the results for the standard cash-in-advance (CIA) constraint. In the second and third columns under the caption DI, the results are derived in the cash-credit economy for two different values of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).