Optimal Taxation under Regional Inequality*

Sebastian G. Kessing\textsuperscript{a} Vilen Lipatov\textsuperscript{b} J. Malte Zoubek\textsuperscript{c}

March 24, 2017

Abstract

Combining an intensive labor supply margin with an extensive, productivity-enhancing migration margin, we determine how regional inequality and labor mobility shape optimal redistribution. We propose the use of delayed optimal-control techniques to obtain optimal tax formulae with location-dependent productivity and two-dimensional heterogeneity. Our baseline simulations using the productivity differences between large metropolitan and other regions in the US indicate that productivity-increasing internal migration can constitute a quantitatively important constraint on redistribution. Allowing for regionally differentiated taxation with location-dependent productivity, we find that marginal tax rates in high- (low-)productivity regions should be corrected downwards (upwards) relative to a no-migration benchmark.

JEL classification: H11, J45, R12

Keywords: Optimal taxation, redistribution, regional inequality, migration, multidimensional screening, delayed optimal control

\textsuperscript{*}We would like to thank David Agrawal, David Albouy, Spencer Bastani, Mireille Chiroleu-Assouline, Bill Hoyt, Christian Gollier, Laurence Jacquet, Luca Micheletto, Casey G. Rothschild, seminar participants in Berlin, Mannheim, Nuremberg, Paris, Trier, and participants of the 11th World Congress of the Econometric Society, CESifo Area Conference in Applied Microeconomics 2015, as well as IIPF 2015 Conference for their valuable comments. Financial support from the Deutsche Forschungsgemeinschaft (DFG), project No. KE 1693/3-1, is gratefully acknowledged.

\textsuperscript{a}University of Siegen and CESifo, kessing@vwl.wiwi.uni-siegen.de
\textsuperscript{b}Compass Lexecon Brussels and CESifo, vlipatov@compasslexecon.com
\textsuperscript{c}University of Siegen, malte.zoubek@uni-siegen.de
1 Introduction

Regional productivity differences are large in many countries. Real per capita GDP in the New England Region was 40% higher than in the Southeast Region in the US in 2015 (BEA, 2017), for example. In Italy, the 2014 real per capita GDP of the Northern and Central Regions was even 77% higher than in the Southern and Islands Region (ISTAT 2017). The spatial dispersion of wages and incomes is well documented, and the underlying causes are still subject to debate (Barro and Sala-i-Martin 1991, Ciccone and Hall 1996, Kanbur and Venables 2005, Acemoglu and Dell 2010, and Young 2013, among others). Given such productivity differences, the efficiency-enhancing potential of interregional mobility is substantial, and increases in personal income are key drivers of this mobility; see Kennan and Walker (2011).

Centralized redistribution schemes such as a federal income tax or federal social transfers reduce interregional migration incentives, since an individual who migrates from a low- to a high-productivity region has to share the realized productivity gains with the government through higher taxes or lower transfers. This generates a trade-off for an inequality-averse policy maker between redistribution and productivity-enhancing interregional migration. Interestingly, this role of internal migration for optimal tax policy has been neglected in the literature, in stark contrast to the role of emigration of high-income earners to low-tax countries or the immigration of welfare recipients from less generous jurisdictions.\footnote{Studies addressing external migration include Mirrlees (1982), Wildasin (1991), Wilson (1992), Lehmann et al. (2014), and others.} We develop a conceptual framework to analyze the implications of internal migration for an optimal tax-transfer policy and assess its quantitative importance. While our focus is on productivity-enhancing migration between regions with permanent productivity differences, our approach may also be used to address the related optimal-taxation problem that arises with respect to migration responses to idiosyncratic shocks to regional labor markets, as discussed by Blanchard and Katz (1992) and Yagan (2014).
We propose a two-dimensional optimal-taxation model which combines an extensive interregional migration decision with an intensive labor supply decision. Our key innovation is the productivity-enhancing nature of the migration margin. The actual or realized productivity of individuals of any given innate productivity is location-dependent, so that individuals can increase their productivity by migrating from a low- to a high-productivity region. Thus, the extensive migration margin also affects the intensive labor supply decision, since productivity and, unless the marginal tax rate is constant, the marginal tax rate change whenever an individual decides to migrate, even if the same tax schedule applies nationwide.

This framework allows us to determine the optimal federal tax schedule as a function of the government’s redistributive preferences, the observed regional earnings distributions, the earnings elasticity, and the distribution of migration costs. Our analysis shows that optimal marginal tax rates tend to be below the benchmark without interregional migration, since the decision to migrate to an area with higher productivity bears a fiscal externality. Its size depends on migration costs, on the income distribution, and on the tax differential, which is itself a function of regional productivity differences and the tax schedule. If marginal tax rates are positive, the fiscal externality is positive, so that optimal marginal tax rates are lower than in a situation without migration. This result is independent of potential cost of living differences between regions, since for the size of the fiscal externality only the foregone tax revenue is decisive. However, for some productivity distribution, the migration opportunity can make negative marginal tax rates optimal. The latter result is similar to other studies that have analyzed the optimal tax-transfer schedule with an extensive participation margin (see, e.g., Saez 2002 and Jacquet et al. 2013).

Our approach provides a methodological contribution to the theory of optimal taxation, in that we endogenize individuals’ type along the productivity dimension through the extensive margin. This is a useful extension to the class of multidimensional screening models, originally discussed by Rochet and Choné (1998) and Armstrong (1996), and further developed to study the taxation of couples.
by Kleven et al. (2009) in the optimal-taxation context. We argue that variants of these models with endogenous productivity can be fruitfully studied using the delayed optimal-control approach as recently formally analyzed by Göllmann et al. (2008) in its entire generality.

Variations of our framework are suitable to address a range of two-dimensional screening problems, where an extensive margin directly affects agents’ productivity (their type), and thus the intensive margin. The decision to participate in the labor market, for example, affects productivity, because non-participation tends to depreciate human capital. Similarly, the decision to switch to a better job also affects actual individual productivity. The same holds true for discrete education or personal decisions such as the decision to attend college, or the decision to have children. All these examples represent empirically relevant dimensions of the labor supply decision. Our framework and the proposed delayed optimal-control technique provide a readily applicable solution for characterizing optimal-taxation schedules and bringing them to data in such cases.

We also study regionally differentiated tax-transfer schemes. To the extent that such schemes are explicit, they are often difficult to enforce in practice in view of the challenge of monitoring the actual place of residence of individuals, and may also be challenged on the grounds of the violation of horizontal equity. Despite these caveats, regional differentiation of labor income taxation can be an element of real-world tax systems. From 1971 to 1994, the German tax system, for example, treated residents in West Berlin differently from people in the rest of the country. Another example is the current path towards a more fiscally integrated Europe. As the EU is moving towards deeper fiscal integration, the option of a regionally differentiated (albeit coordinated) tax transfer remains an alternative to uniform EU-wide taxation (e.g., Lipatov and Weichenrieder 2016). This decision requires understanding of the advantages and the challenges of a differentiated system vis-à-vis an integrated system.\footnote{To the extent that the Member States are unrestricted by the center in deciding on their own tax-transfer schemes, additional considerations of tax competition have to be taken into account. See Lehmann et al. (2014) for the analysis of such considerations in the optimal-taxation framework. Bargain et al. (2013) have contrasted a Member-States-based redistribution scheme}
income taxation amounts to regionally differentiated taxation in real terms, due to cost-of-living differences; see Albouy (2009). We show that within our optimal-taxation framework with productivity-increasing migration, the migration effect exerts additional downward (upward) pressure on differentiated marginal tax rates in the high- (low-)productivity region. This is the opposite of the implied differentiation resulting from cost-of-living differences and nominally undifferentiated taxation.

Conceptually, we add to the debate on tagging in optimal taxation by considering the region of residence as an endogenous tag. In contrast to the existing literature, our tag can be used to encourage productivity-enhancing migration and not only to relax information constraints. Our analysis therefore sheds light on the question how optimally differentiated taxation should trade-off enhanced redistribution and the potential fiscal externality from productivity-enhancing migration.

The next section discusses the related literature. Section 3 introduces the theoretical framework, and we derive our theoretical results for unified and differentiated taxation in Sections 4 and 5, respectively. We present a numerically calibrated illustrative simulation based on US micro data in Section 6 and leave the proofs to Appendices. A further Supplement is also available separately.

2 Related literature

The normative implications of productivity-enhancing internal migration for optimal redistribution have, to the best of our knowledge, not been studied to date. The constraint of interjurisdictional or international mobility for the redistribution policy of a single jurisdiction or country, however, has received considerable attention within the optimal-taxation literature and beyond; see, in particular, Mirrlees (1982), Wildasin (1991), Wilson (1992), Simula and Trannoy (2011), Lehmann et al. (2014), and Lipatov and Weichenrieder (2015), among others. We show that labor mobility within a sufficiently large jurisdiction or between regions within a

with an integrated scheme in Europe. However, they address the implications for macroeconomic stabilization, whereas we study the efficiency of redistribution.
country can also be important for redistribution.

Conceptually, our analysis belongs to a class of two-dimensional screening models that have been recently used to analyze a range of tax policy questions. Lehmann et al. (2014) combine the intensive labor supply margin with an extensive migration margin. Their focus is on independent governments competing for internationally mobile high-productivity individuals, and it is therefore complementary to our analysis of optimal taxation by a single government. Moreover, individual productivity is not location-dependent in their analysis, and they only focus on the threat of migration, whereas actual, productivity-enhancing migration is at the heart of our approach. Gordon and Cullen (2012) also use an optimal-taxation approach to study interregional migration in a model with several states. However, they focus on the assignment problem of whether redistribution should be carried out at the national or the subnational level and do not consider productivity differences. Using a fiscal federalism setting, Blomquist and Micheletto (2009) identify advantages of redistributive in-kind transfers carried out at the regional level in presence of a national but uniform income tax system. Jacquet et al. (2013) also study a two-dimensional optimal-taxation model but focus on participation.

The structure of our approach owes much to Kleven et al. (2006, 2009), who study the optimal taxation of couples with cooperative households. Their analysis combines the intensive labor supply decision with the household’s choice to become a single- or a double-earner household. However, our analysis differs in several important ways from their framework. First, we consider individuals and not households consisting of two persons whose incomes may be taxed separately. Secondly, in our approach, individuals originally reside in different regions, so they differ not only in costs of changing the location, but also in the group they originally belonged to. Finally and most importantly, we introduce a link between endogenous individual productivity and the decision along the extensive margin, and we show how models featuring this link can be solved using delayed optimal control.
Rothschild and Scheuer (2013, 2016) and Gomes et al. (2017) also study optimal taxation of rent-seeking activities and optimal taxation in the Roy model\(^3\), using two-dimensional screening approaches. Wages are endogenously determined in their work, either by the total labor supply in a given sector, or by total rent-seeking activities. Similarly, Scheuer (2014) studies entrepreneurial taxation with an endogenous decision of whether to become an entrepreneur or a worker, where these decisions determine relative compensation in the aggregate. In our study, individual productivity and thus market compensation depend directly on the discrete decision of individuals and not on aggregate outcomes. Accordingly, our argument for optimally adjusting marginal tax rates is not based on the attempt to manipulate relative wages, but on the wish to encourage productivity-enhancing regional mobility given the associated fiscal externality under progressive income taxation.

Our analysis of regionally differentiated taxation relates to the increased interest in tagging in the design of tax-transfer schemes. The idea that the government’s information problem can be relaxed by using additional observable characteristics (tags) that are correlated with individual productivity goes back to Akerlof (1978) and has recently been discussed intensively in the optimal-taxation literature; see Immonen et al. (1998), Boadway and Pestieau (2006), Blomquist and Micheletto (2008), Cremer et al. (2010), Mankiw and Weinzierl (2010), Weinzierl (2011), Bastani et al. (2013), and Best and Kleven (2013).\(^4\) We add to this literature in several ways. First, we consider the region of residence as a potential tag. Secondly, we explicitly study a tag that is endogenous and can be adjusted by individuals subject to some cost. Moreover, changing the tag directly affects productivity. In this respect, our paper is related to the literature that studies the interplay between human capital formation and optimal taxation, where the former shapes the productivity distribution and the latter influences incentives for

\(^3\)Note that the Roy model as formulated in these papers is not perfectly suited for modeling migration decision, as individuals are born into a certain region - a constraint absent in the sectoral choice decision.

\(^4\)Golosov et al. (2014) provide a general framework to study multidimensional optimal taxation using a variational approach.
human capital formation; see Krueger and Ludwig (2013) and Stantcheva (2015) and the references therein. The endogeneity of productivity also relates our work to that of Best and Kleven (2013), who consider a dynamic setting where individual productivity depends on the previous intensive labor supply decisions.

Finally, Eeckhout and Guner (2017) also study the effects of a progressive federal income tax on the spatial allocation of economic activity with a heterogeneous population, and also consider regionally differentiated taxation, but they do not use a Mirrleesian optimal-taxation framework and do not consider the interaction of the intensive labor supply decision and the interregional migration decision.

3 The framework

We consider two sources of heterogeneity across workers: innate productivity $n$ and migration costs $q$. These original individual characteristics are distributed over $[n_{\text{min}}, n_{\text{max}}] \times [0, +\infty)$, and the government can observe neither of them. There are two regions, $i = A, B$, with total population normalized to two. Originally, half of the population resides in each region, but the endogenous migration decisions of individuals change these population shares. Our key assumption is that the regions differ in their productivity. An individual’s actual or realized productivity $n_i$ is a function of her innate productivity and her region of residence: $n_i = \omega(n, i) = \omega_i(n)$, where $\omega_i$ is strictly increasing in $n$. We normalize $n_A = \omega_A(n) = n$. Accordingly, the function $n_B = \omega_B(n) = \omega(n)$ not only assigns the actual productivity to all original residents of region $B$, but also indicates the transformation of productivity for individuals who migrate from $A$ to $B$. Without loss of generality we assume that region $B$ is the more productive region, so that $\omega(n) > n$. Innate productivity is distributed in each region $i$ according to the unconditional probability distribution $f(n)$ on $[n_{\text{min}}, n_{\text{max}}]$.\footnote{It is straightforward to extend the analysis to the case in which regions also differ in their distribution of innate productivity. Similarly, we could allow for negative migration costs for some subset of individuals at each innate productivity level without affecting the results qualitatively. The latter can generate migration in both directions. For clarity, we abstract from these further aspects.}
optimal-taxation literature, we treat wages as exogenous and independent of individual labor supply and aggregate migration decisions. Accordingly, the analysis applies to a situation where the effect of migration flows on wages is negligible. The empirical evidence supports the view that, for sufficiently large regions, the effects of internal migration on wages are rather small; see, for the US, Boustan et al. (2010), and for evidence from the German reunification, D’Amuri et al. (2010) and Frank (2009).6

Following Diamond (1998), we use preferences that are separable in consumption and labor. The utility function of a worker of type \((n, q)\) is similar to the formulation in Kleven et al. (2009), but depends on the region of residence:

\[
  u(c, z, l) = c_i - n_i h \left( \frac{z_i}{n_i} \right) - q^c l + q^h (1 - l),
\]

where \(l\) is an indicator variable that takes the value 1 in case of migration.7 The function \(h(\cdot)\) is increasing, convex, and twice differentiable. It is normalized so that \(h'(1) = 1\) and \(h(0) = 0\). The other variables have standard interpretations. Consumption \(c_i\) equals gross income \(z_i\) minus taxes \(T_i\), the latter depending on gross income: \(c_i = z_i - T_i(z_i)\). Total migration costs are potentially made up of two components: \(q = q^c + q^h\), where \(q^c\) is the cost of moving (the need to adapt to new conditions, to learn a new language on moving between regions where different languages are spoken, the transaction costs of selling your old house and buying a new one, etc.), and \(q^h\) is the utility derived from being at home and benefiting from the existing social networks. To isolate the effects of the two types of heterogeneity, it is useful to consider them separately. The pure cost-of-moving model sets \(q = q^c\) and \(q^h = 0\); the pure home-attachment model uses \(q = q^h\) and \(q^c = 0\). Ex post (i.e., after migration has taken place), heterogeneity in \(q^c\) reflects the differences between individuals who migrate, whereas heterogeneity in \(q^h\) reflects the differences between individuals who stay in their home region. In what

---

6For similar findings in the case of immigration of foreigners see Borjas (1994) and Ottaviano and Peri (2007, 2008).

7Note that in this and following expressions, we usually drop the argument of \(z_i(n_i)\) for parsimony, but \(z_i\) is understood as the income obtained by the individual of innate productivity \(n\) in territory \(i\).
follows we focus on the cost-of-moving case, but, with some minor modifications, the home-attachment case is quite analogous. However, our optimal tax schedules and their derivations are sufficiently general to encompass both cases.

Each individual chooses $l$ and $z_i$ to maximize (1) for a given tax schedule, i.e., she decides whether to move or not and determines her gross earnings, given that she resides in region $i$. The first-order condition for gross earnings is

$$h'(\frac{z_i}{n_i}) = 1 - \tau_i(z_i),$$

(2)

where $\tau_i$ is the marginal tax rate. Accordingly, $n_i$ can be interpreted as potential income, in that individuals facing a marginal tax rate of zero would realize this level of gross earnings. The elasticity of gross earnings with respect to net-of-tax-rate is defined as

$$\varepsilon_i \equiv \frac{1 - \tau_i}{z_i} \frac{\partial z_i}{\partial (1 - \tau_i)} = \frac{n_i h'(\frac{z_i}{n_i})}{z_i h''(\frac{z_i}{n_i})},$$

and is a function of gross earnings and the region of residence. Finally, we require the following property:

**Assumption** The function $x \rightarrow \frac{1 - h'(x)}{x h''(x)}$ is decreasing.

Consider now the migration decision. We denote by $p(q|n)$ the density of $q$ conditional on $n$, and by $P(q|n)$ the cumulated distribution of $q$ conditional on $n$. Conditional on residing in region $i$, the individuals’ choice of gross earnings is determined by (2), which allows us to define indirect utility conditional on the place of residence and net of the migration costs as

$$V_i(n_i) = z_i - T_i(z_i) - n_i h\left(\frac{z_i}{n_i}\right).$$

Individuals will move from $i$ to $j$, $j = A, B$, $i \neq j$, whenever their migration costs are below the net gain from moving, so that $\bar{q}_i(n) \equiv \max \{V_j(n_j) - V_i(n_i), 0\}$ is the critical level of migration costs that determines the actual number of migrants.
for any innate productivity level.

### 3.1 The government’s optimal-tax problem

The government wants to maximize the social welfare function

\[
\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} \Psi \left( V_i(n_i) - q^i l + q^h (1 - l) \right) p(q|n) f(n) dq dn,
\]

where \(\Psi(.)\) is a concave and increasing transformation of individual utilities. Denoting by \(E\) the exogenous expenditure requirement, it needs to respect the budget constraint

\[
\sum_i \int_{n_{\min}}^{n_{\max}} \int_0^{+\infty} T_i(z_i)p(q|n) f(n) dq dn \geq E.
\]

Moreover, the government’s tax schedule needs to be incentive-compatible. Denoting by a dot above a variable its derivative with respect to \(n\), this implies

\[
\dot{V}_i(n) = \left[ -h \left( \frac{z_i}{n_i} \right) + \frac{z_i}{n_i} h' \left( \frac{z_i}{n_i} \right) \right] \omega_i'(n) \geq 0.
\]

Moreover, in case of nondifferentiated taxation, \(T_A(z) = T_B(z)\). We show in the Supplement that a path for \(z_A\) and \(z_B\) can be truthfully implemented by the government using a nonlinear tax schedule.

Let \(\lambda > 0\) be the multiplier associated with the budget constraint (4). The government’s redistributive tastes may be represented by region-dependent social marginal welfare weights. In terms of income, our welfare weights will take the form

\[
g_i(z) = \frac{\Psi'(V_i(z)) (1 - P(\bar{q}_i|z)) + \int_{\bar{q}_i}^{\bar{q}_j} \Psi'(V_i(z) - q^c) p(q|z) dq}{\lambda (1 + P(\bar{q}_j|z) - P(\bar{q}_i|z))}
\]

for the cost-of-moving model, where \(\bar{q}_i(z) \equiv \max \{V_j(z) - V_i(z), 0\}\).

### 4 Optimal unified taxation

We first investigate the optimal nondifferentiated tax-transfer system. The government maximizes (3) subject to (4) and (5) through its choice of \(T(z)\). This
problem formally amounts to a delayed optimal-control problem such as has been analyzed by Göllmann et al. (2008) in its entire generality. In our model, though, the delay is a nonfixed lag, given that we do not require the productivity gain from moving to be constant. The necessary conditions for optimal control in such a setting are presented in Abdeljawad et al. (2009). We describe in Appendix A how the delayed optimal-control approach can be applied to solve the optimal-taxation problem, and we derive all our results rigorously there. Below, however, we follow the intuitive perturbation approach pioneered by Piketty (1997) and Saez (2001) to derive the optimal tax scheme. This heuristic derivation disentangles the economic forces that determine the shape of the optimal marginal tax rate schedule, including the effects generated by productivity-enhancing migration.

We denote the endogenous distribution of gross incomes in both regions by \( v_i(z_i) \), and we denote by \( k \) the endogenously defined function that maps gross income in the low-productivity region to the gross income this individual would earn in the high-productivity region, given his innate productivity and the respective tax treatment, i.e., \( z_B = k(z_A) \).\(^8\) We consider an increase in taxes for all individuals above gross income \( z \). The increase is engineered through an increase in the marginal tax rate \( d\tau \) in the small band \((z, z + dz)\), such that for all individuals with gross earnings above \( z \) the tax payments increase by \( dzd\tau \). This tax increase gives rise to three different effects.

**Revenue effect** All taxpayers in either region with gross incomes above \( z \) pay additional taxes of \( dzd\tau \). The net welfare effect of this tax payment for an affected individual in region \( i \) with gross earnings \( z' \) is given by \( dzd\tau \left( 1 - g_i(z') \right) \), and the total effect is then

\[
R = dzd\tau \int_z^{\infty} \left\{ [1 - g_A(z')] v_A(z') s_A(z') + [1 - g_B(z')] v_B(z') s_B(z') \right\} dz',
\]

where \( s_A(z) = 1 - P(\hat{q}_A|z) \) and \( s_B(z) = 1 + P(\hat{q}_A|k^{-1}(z)) \).

**Behavioral effect** Individuals in the band \((z, z + dz)\) will change their labor

\(^8\)In terms of our previous formulation, an individual of ability \( n \) receives gross income \( z_A = z(n) \) in region A and gross income \( z_B = z(\omega(n)) \) in region B, where this notation abstracts from the fact that the gross income also depends on the tax schedule.
Figure 1: The migration effect comes into play for individuals for which $z_A' < z$ and $z_B' \geq z$.

supply in response to the increase in the marginal tax rate. Given that $\varepsilon \equiv \frac{1-\tau}{z} \frac{dz}{d(1-\tau)}$, each individual in the band will reduce its income by $-d\tau \varepsilon \frac{z}{1-\tau}$. There are approximately $dz [v_A(z)s_A(z) + v_B(z)s_B(z)]$ of these individuals. To take the non-linearity of the tax schedule into account, we explicitly correct this with the "feedback" effect $1 + \frac{\tau'}{1-\tau} z\varepsilon$ following Golosov et al. (2014). The total effect on tax revenue is

$$L = -d\tau dz \frac{\tau z\varepsilon}{1-\tau} [v_A(z)s_A(z) + v_B(z)s_B(z)] \left(1 + \frac{\tau'}{1-\tau} z\varepsilon\right).$$

Migration effect An increase in taxes for all individuals above gross income $z$ does not affect the migration decision of individuals with gross income $z_A' \geq z$, and accordingly also $z_B' > z$, such that the tax increase affects them in both regions alike. The same holds true for all individuals for which $z_B' = k(z_A') < z$ and accordingly $z_A' = k^{-1}(z_B') < z$. However, as illustrated in Figure 1, for all individuals for which $z_A' < z$ and $z_B' \geq z$ the migration decision is negatively affected. In this range, all individuals whose cost of moving is between $\bar{q} - dz\tau d\tau$ and
$q$ will now decide not to migrate. There are $p(q|z) v_A(z) dz d\tau$ affected individuals at any concerned level of income, with a resulting tax effect of $T_A(z) - T_B(k(z))$ for each of them. The total migration effect is thus

$$M = d\tau dz \int_{\hat{z}} [T(z') - T(k(z'))] p(q'|z') v_A(z') dz',$$

where $\hat{z} \equiv k^{-1}(z)$. Note that there is an endogenous effect on the income distribution in each region. This effect does not come into play explicitly here, since we express the effects in terms of the posterior distribution. The three effects must balance out in the optimum: $R + L + M = 0$. From this we have our first result.

**Proposition 1** The optimal unified tax schedule is characterized by

$$\frac{\tau}{1-\tau} = \mathcal{A}(z) \mathcal{B}(z) [\mathcal{C}(z) + \mathcal{D}(z)],$$

where

$$\mathcal{A}(z) \equiv \frac{1}{\hat{z}} \frac{1-\tau}{1-\tau + \tau' \hat{z}}, \quad \mathcal{B}(z) \equiv \frac{1}{z (v_A(z) s_A(z) + v_B(z) s_B(z))},$$

$$\mathcal{C}(z) \equiv \int_{\hat{z}} \left\{ [1 - g_A(z')] v_A(z') s_A + [1 - g_B(z')] v_B(z') s_B \right\} dz',$$

$$\mathcal{D}(z) \equiv \int_{\hat{z}} [T(z') - T(k(z'))] p(q'|z') v_A(z') dz'.$$

**Proof.** This follows from the exposition above. The equivalence to the optimal, tax formula formally derived by using the delayed optimal-control technique is presented in Appendix A.

It is straightforward to compare the result with the alternative benchmark without migration. The optimal tax schedule then follows the usual Diamond (1998) and Saez (2001) results for the earnings distribution in the entire country without a migration effect. In this case, optimal marginal tax rates are determined by

$$\frac{\tau}{1-\tau} = \mathcal{A}(z) \mathcal{B}(z) \mathcal{C}(z).$$

With $\mathcal{D}(z) < 0$, the disincentive effects of higher tax rates on productivity-increasing mobility tend to reduce marginal tax rates, but note that $\mathcal{B}(z)$ and $\mathcal{C}(z)$ are endogenously determined by the migration flows, so that (6) and (7)
cannot be directly compared in general. To make the result more formal, consider the following proposition:

**Proposition 2** If marginal tax rates are positive, a government taking the effect of taxes on the migration decision into account sets lower marginal tax rates than a government which disregards the migration decision, but faces the same posterior income distribution generated by migration.

**Proof.** See appendix.

Note that positive marginal tax rates are a sufficient but not a necessary condition for this result. Whenever $\mathcal{D} < 0$ for any given level of gross income $z$, marginal tax rates are lower with migration than for the no-migration benchmark with the posterior distribution. Thus, whenever productivity-enhancing migration implies a positive fiscal externality at a given innate productivity level, marginal tax rates should be reduced to take the fiscal externality of interregional migration appropriately into account. This constrains optimal redistribution beyond the classic adverse labor supply responses.

Another direct implication of the optimal unified taxation formula (6) is stated in the following proposition.

**Proposition 3** Optimal marginal tax rates can be negative.

**Proof.** For $\mathcal{D}(z) < 0$, it is possible that $\mathcal{C}(z) + \mathcal{D}(z) < 0$, and thus $\tau < 0$.\(^9\)

Similarly to the findings of other studies that combine an extensive participation decision with the intensive labor supply decision, our setting potentially gives rise to negative marginal tax rates.

Another benchmark to compare our optimal solution with is an economy with the ex ante distribution of productivity and without internal migration. As we show in the Appendix, the comparison of optimal marginal tax rates is less clear-cut in this case. The endogeneity of the posterior productivity distribution when allowing for migration also affects $\mathcal{B}(z)$ and $\mathcal{C}(z)$, and these effects may drive

---

\(^9\)By simulative example (available upon request) it can be shown that this is the case for certain parameter values.
optimal marginal tax rates in the opposite direction. Formally, we provide a sufficient condition for mobility to decrease the marginal tax rates for this alternative benchmark in Appendix AA.

Finally, we make the following remark about the welfare comparison in the unified taxation case, highlighting the desirability of productivity-enhancing migration.

**Remark** The welfare achieved with unified taxation in the no-migration case is not higher than the welfare achieved with migration.

**Proof.** Consider the tax schedule that maximizes welfare if migration is not allowed. Migration brings a Pareto improvement, because individuals move only if they find themselves better off. Furthermore, with migration to the richer region only, the government budget constraint will not be violated, since the tax schedule is nondecreasing in income without migration. Thus, under the same tax schedule the welfare cannot decrease with the introduction of a migration possibility. Finally, the government will change the tax schedule only if it brings a further increase in welfare. Thus, the welfare with migration cannot be lower than welfare with no migration. ■

5 Optimal differentiated taxation

We now consider the possibility that the central government can choose differentiated tax schedules for both regions. If there were regional productivity differences but no migration, this setting would correspond to the analysis of an optimal tax scheme with tagging on the region of residence. However, we continue to assume that migration between the regions is possible, that productivity is location-dependent, and that individuals are heterogeneous with respect to their migration costs, which are unobservable by the government. Again we employ the perturbation approach and delegate the formal proofs to Appendix A.

We first study the optimal tax schedule in the low-productivity region. Consider an increase of taxes in region $A$ for all individuals above gross income $z_A$. 
The increase is engineered through an increase in the marginal tax rate $d\tau_A$ in the small band $(z_A, z_A + dz_A)$, such that all individuals with gross earnings above $z_A$ increase their tax payments by $dz_A d\tau_A$. This generates three effects.

**Revenue effect:** All taxpayers in $A$ with incomes above $z_A$ pay additional taxes of $dz_A d\tau_A$. The net welfare effect of this tax payment for an individual with gross earnings $z_A'$ is given by $dz_A d\tau_A (1 - g_A(z_A'))$, and the total effect is

$$R_A = dz_A d\tau_A \int_{z_A}^{\infty} [1 - g_A(z_A')] v_A(z_A') s_A(z_A') dz_A'.$$

**Behavioral effect:** Individuals in the band $(z_A, z_A + dz_A)$ will change their labor supply in response to the increase in the marginal tax rate. Given that

$$\varepsilon \equiv \frac{1 - r_i}{z_i} \frac{dz_i}{d(1 - r_i)},$$

each individual in the band will reduce its income by $-d\tau_A \varepsilon \frac{z_A}{1 - \tau_A}$. There are approximately $dz_A v_A(z_A) s_A(z_A)$ of these individuals, so that the total effect on tax revenue, taking into account the nonlinearity correction term $1 + \frac{\tau_A' z_A \varepsilon}{1 - \tau_A}$ is

$$L_A = -d\tau_A dz_A \varepsilon \frac{\tau_A}{1 - \tau_A} z_A v_A(z_A) s_A(z_A) \left(1 + \frac{\tau_A' z_A \varepsilon}{1 - \tau_A}\right).$$

**Migration effect:** An increase in taxes for all individuals above gross income $z_A$ affects the migration decision of individuals with gross income in region $A$ above this level. At any income level $z \geq z_A$ individuals whose cost of moving is between $\bar{q}$ and $\bar{q} + dz_A d\tau_A$ will now decide to migrate. There are $p(\bar{q} | z_A) v_A(z_A) dz_A d\tau_A$ affected individuals, with a resulting tax effect of $T_B(k(z_A)) - T_A(z_A)$ for each of them. If the schedule results in migration from region $B$ for people of income $z$, the argument is analogous, as we show formally in the Appendix. The total effect is thus

$$M_A = dz_A d\tau_A \int_{z_A}^{\infty} [T_B(k(z_A')) - T_A(z_A')] p(\bar{q} | z_A') v_A(z_A') dz_A'.$$

In the optimum, these effects should cancel out, so that optimal marginal tax rates
can be characterized by

\[
\frac{\tau_A}{1 - \tau_A} = \frac{1}{\varepsilon z_A v_A(z_A) s_A(z_A) 1 - R_A + \tau'_A z_A \varepsilon_A} \times \int_{z_A}^{\infty} \{[1 - g_A(z'_A)] s_A(z'_A) + [T_B(k(z'_A)) - T_A(z'_A)] \rho(q | z'_A)\} v_A(z'_A) dz'_A.
\] (8)

We turn now to the optimal tax schedule in the high-productivity region. We consider a small increase in taxes by \(dz_B d\tau_B\) for all individuals above \(z_B\) in region \(B\). This again generates three effects, which must balance out along the optimal tax schedule, so that

\[
\frac{\tau_B}{1 - \tau_B} = \frac{1}{\varepsilon z_B v_B(z_B) s_B(z_B) 1 - \tau_B + \tau'_B z_B \varepsilon_B} \times \int_{z_B}^{+\infty} \{[1 - g_B(z'_B)] s_B(z'_B) - [T_B(z'_B) - T_A(z^{-1}(z'_B))] \rho(q | z'_B)\} v_B(z'_B) dz'_B.
\] (9)

Both optimal tax schedules are derived rigorously in Appendix A. The optimal tax formulae not only differ in the average welfare weights and in the corresponding productivity distributions above the relevant gross income level, but they also take the fiscal migration externality into account. Typically, this externality will be negative for the high-productivity region and positive for the low-productivity region. Accordingly, from these optimal tax schedules (8) and (9) we have the following result.

**Proposition 4** For all levels of innate productivity and the corresponding gross incomes, the marginal tax rate \(\tau_A\) in the low-productivity region is increasing in the difference in total tax liability between the high- and the low-productivity regions, and the marginal tax rate \(\tau_B\) in the high-productivity region is decreasing in this difference in total tax liability.

**Proof.** The result follows directly from (8) and (9). 

Intuitively, the larger the potential fiscal gains are from working in the high-productivity region instead of working in the low-productivity region, the more the government distorts labor supply in the low-productivity region and the less
it distorts labor supply in the high-productivity region. In the Supplement we additionally rearrange the formulae (8) and (9) to show how the migration semielasticities act as a correction factor to the region-dependent marginal social welfare weights to determine optimal marginal tax rates.

Next we consider the asymptotic properties of the differentiated tax schedule. Suppose the distribution of innate ability, \( f(n) \), has an infinite tail (\( n_{\text{max}} = \infty \)). As is standard in the literature, we assume that \( f(n) \) has a Pareto tail with parameter \( a > 1 \) (\( f(n) = C/n^{1+a} \)). Moreover, we assume that \( P(q|n), T_B-T_A, \tau_A, \tau_B, \bar{q}_A, \bar{q}_B \) converge to \( P^\infty(q), \Delta T^\infty, \tau_A^\infty < 1, \tau_B^\infty < 1, \bar{q}_A^\infty, \bar{q}_B^\infty \) as \( n \to \infty \). Finally, we concentrate on the case where, for sufficiently large \( n \), we have \( \omega(n) = n + c \), where \( c \geq 0 \) is a finite constant. In this case, the following proposition arises:

**Proposition 5** Under the assumptions on convergence formulated above, (i) the average marginal social welfare weights in the two regions converge to the same value \( \bar{\psi}/\lambda \geq 0 \); (ii) the difference between the taxes in the two regions converges to zero, \( \Delta T^\infty = 0 \); and (iii) the marginal tax rate in both regions converges to \( \tau^\infty \) with

\[
\frac{\tau^\infty}{1-\tau^\infty} = \frac{1}{a \varepsilon^\infty} \left( 1 - \frac{\bar{\psi}}{\lambda} \right).
\]

**Proof.** See Appendix AA. ■

The intuition for zero difference of top taxes is similar to that in Kleven et al. (2009). Starting from a wedge between \( T_B \) and \( T_A \), say, with \( T_B > T_A \), welfare could be increased by marginally reducing this wedge due to the migration effect. In particular, consider a reform that increases \( T_A \) and decreases \( T_B \) above some \( n \), while keeping the tax revenue constant in the absence of migration. The direct welfare effects of such a reform cancel out, because \( g_A \) and \( g_B \) have converged to \( g^\infty \). The fiscal effects due to the earnings responses cancel out as well, because \( \Delta T^\infty \) is constant and thus the labor supply elasticities are identical. The fiscal effect due to migration is however positive, because some people would move to region B and pay higher taxes. If the starting wedge is characterized by \( T_B < T_A \), the welfare can be improved by the opposite reform. Thus, though there are
substantial differences between a differentiated and a unified tax schedule, they
disappear in the limit at the top of the distribution.

Finally, we consider how tax rates should be differentiated if a full differentia-
tion of the entire tax schedule is not feasible. First, we study a tax system that is
separable in the sense that the same income faces the same marginal tax in both
regions, but allowing for region-dependent total tax liabilities. The maximization
problem of the government is as before, except that instead of the restriction that
\( \Delta T = 0 \) we have \( \Delta T = C \), where \( C \) is constant and \( \Delta T := T_B - T_A \). For this
setting, we find:

**Proposition 6** Starting from a unified taxation schedule in the two regions, if the
government is allowed to make a lump-sum transfer between regions, it will choose
to make a transfer from the less productive to the more productive region.

**Proof.** See Appendix AA. ■

While this result may appear surprising, there is a clear economic intuition
behind it. The tax on the poor region has to be higher in order to induce extra
migration, which is productivity-enhancing. The extensive margin is used to in-
crease efficiency via increased labor mobility, whereas redistribution is engineered
through the intensive margin. This result is independent of the interpretation of
migration costs.

Moreover, consider a tax schedule that is separable in the sense that \( \tau_A = \tau_B \).
Starting from this schedule, the following proposition shows that in the cost-of-
moving model decreasing the marginal tax in region B and increasing it in region
A would be desirable:

**Proposition 7** If \( \Psi' \) is convex, \( q \) and \( n \) are independently distributed, and \( \omega'(n) \geq 1 \), then it is optimal to introduce some wedge in marginal taxes into the system of
separable taxation of the two regions. In particular, in the cost-of-moving model it
is optimal to decrease the marginal tax in the high-productivity region and increase
it in the low-productivity region.

**Proof.** See Appendix AA. ■
The proof is based on the fact that in the cost-of-moving model the difference in marginal welfare weights of residents of regions A and B is decreasing in productivity, if the social welfare exhibits prudence (marginal social welfare is convex). Thus, it makes sense to make the lower part of the productivity distribution in region A marginally happier than in region B, while making the upper part of productivity distribution in region B marginally happier than in region A. Hence, lower marginal tax rates in region B are optimal. The productivity transformation function \( \omega(n) \) may however reverse this finding, if migration in the lower part of the distribution is subject to substantially larger productivity gains than migration in the upper part of the distribution, i.e., \( \omega'(n) < 1 \); hence the condition on this function.

6 Simulation and Calibration

In this section we provide numerical simulations for the US, to gain insights into the quantitative importance of productivity-enhancing migration for the design of tax policy and optimal redistribution.\(^{10}\) We first focus on the difference between an optimal unified tax schedule with and without migration for a given posterior productivity distribution as in Propositions 1 and 2. To apply our framework empirically, we divide the US into a high-productivity region (large metropolitan) and a low-productivity region (other). We use the observable income distribution to recover the underlying productivity distributions in both regions, as well as the implied migration gains for workers of different innate productivity. We then simulate the unified optimal tax formulae with and without productivity-enhancing migration for the posterior productivity distribution to gauge the difference between them. Finally, we also simulate the optimal differentiated tax schedules for the high- and low-productivity regions. In what follows, we first specify functional forms and parameters used in the simulations, and then describe the calibration.

\(^{10}\)Our simulations use a modified and extended version of the code developed by Henrik Kleven, Claus Thustrup Kreiner, and Emmanuel Saez applied in Kleven et al. (2009). We would like to express our gratitude to them for providing their original code to us.
6.1 Simulation specification

For simulations we use isoelastic utility \( h(\frac{z}{n}) = (\frac{z}{n})^{1+\varepsilon}/(1+\varepsilon) \) with a constant earnings elasticity \( \varepsilon = \frac{1}{\varepsilon} \) as in Saez (2001). Paralleling our theoretical derivations, we concentrate on the cost-of-moving model; hence \( q = q^c \). Moreover, we follow Kleven et al. (2009) by assuming a power law distribution for the costs at the extensive margin on the interval \([0,q_{\text{max}}]\) with \( P(q) = (q/q_{\text{max}})^\eta \) and \( p(q) = (\eta/q_{\text{max}}) \cdot (q/q_{\text{max}})^{\eta-1} \). This distribution of \( q \) is the same in each region and independent of \( n \), that is, \( \partial q_{\text{max}}/\partial n = 0 \). The parameter \( \eta \) may be interpreted as a migration elasticity of the form \( \eta = \frac{q}{P(q/n)} \frac{\partial P(q/n)}{\partial q} = \frac{\partial P(q/n)}{P(q/n)}. \) As the social objective function, we use the constant rate of risk aversion (CRRA) function \( \Psi(V) = V^{1-\gamma}/(1-\gamma) \), where the parameter \( \gamma \) measures the government’s preference for equity. We choose \( \gamma = 1 \); hence \( \Psi(V) = \log(V) \) in line with Chetty (2006).

6.2 Calibration to the US

We proceed with the calibration of our economy to the US in four steps. First, we choose regions by focusing on the considerable productivity discrepancy between regions with different levels of urbanization in the US. To do this, we draw on the Rural Urban Continuum Code (RUCC, also known as the Beale code) that is provided by the US Department of Agriculture. The RUCC assigns each county to one of nine classes. Starting with highly urban counties central in a metropolitan area and with a population of more than 1 million (class 1), the code goes up to 9 for completely rural counties that are not adjacent to a metropolitan area and/or exhibit a population of less than 2,500. The Panel Study of Income Dynamics (2013, PSID) provides the RUCC for each individual’s county of residence. We define all counties belonging to class 1 as the large metropolitan areas (region B), and counties of classes 2 through 9 as other areas (region A), as illustrated by Figure 2.

Second, we recover the ability distributions for these regions using individuals’ maximization as given by Equation (2) with earnings elasticity \( \varepsilon = 0.25 \) as sug-
gested by Saez (2001). Specifically, we combine the 2008 individual gross labor income data from the 2009 PSID for working head of the househodles with the corresponding marginal tax rate from the NBER TAXSIM model.\textsuperscript{11} This procedure is similar to that of Best and Kleven (2013), who differentiate individuals by age, whereas we use a regional distinction. As suggested by Diamond (1998) and Saez (2001), very high incomes are well approximated by a Pareto distribution. Therefore, the skill distributions are modified by assuming a Paretian shape for each top 5\% of the respective distribution corresponding to abilities above \$184,717 (\$130,271) in the large metropolitan (other) areas, and we estimate the specific Pareto parameter for both regions.\textsuperscript{12}

The procedure to recover the productivity distribution is as follows: Each individual’s innate ability is computed from the individual utility maximization, using her income data from the PSID, and the actual marginal tax rate corresponding to this income level from TAXSIM together with the earnings elasticity $\varepsilon$ and the functional assumption for $h(\frac{z}{n})$. For all potential incomes above \$184,717

\textsuperscript{11} The NBER TAXSIM model v9 is applied, see http://www.nber.org/taxsim/.

\textsuperscript{12} Denoting by $n_m$ the average ability above, or equal to, ability $n$, the estimation of the respective Pareto parameter is conducted by regressing $n_m/n$ on a constant in the interval of ability levels between the top 10\% and the top 5\%. This interval corresponds to ability levels between \$135,699 (\$93,487) and \$184,717 (\$130,271).
Figure 3: Revealed true abilities (plot A) and revealed lag function \( \kappa(n) \) (plot B) for the two chosen regions of the US, based on data from PSID 2008/09 and TAXSIM. Revealed abilities are continously smoothed.

($130,271), the computed ability is then replaced by the respective Pareto value. Figure 3A depicts the computed skill distributions in regions A and B. The resulting descriptive statistics for both areas exhibit a median ability difference of 17.1% and a mean difference of 38.5% between the large metropolitan and other areas.

Third, we estimate the transformation function \( \omega(n) \) from the regional ability distributions. We compute the difference in the mean ability in each per-mille of the distribution. This difference is assumed to be the productivity increase for the mean person (sampling point) of each per-mille. The lag function \( \kappa(n) = \omega(n) - n \) is then estimated by linear interpolation using the sampling points, and it is presented in Figure 3B.\(^{13}\) We find a substantial productivity increase from migration to a large metropolitan district from elsewhere, in particular for the types with potential annual incomes of around $100,000–$250,000 at their origin. For high ability levels (above $250,000) we make the additional assumption that \( \kappa(n) \) is constant, because of the lack of sampling points in this range. The cutoff productivity level of $250,000 implies that we use more than 99% of the total mass of the productivity distribution.

Fourth, we calibrate the migration cost distribution using the migration elas-

\(^{13}\)The Supplement provides a detailed justification of our empirical construction of \( \omega \).
ticity \( \eta \) and the parameter \( q_{\text{max}} \). We use \( \eta = 1.5 \), and additionally consider lower and higher values in a range between \( \eta = 0.2 \) and \( \eta = 2 \) to assess the sensitivity of the results to this parameter, since there are no reliable estimates for the migration elasticity between more and less urbanized regions. We choose \( q_{\text{max}} \) such that the average moving costs across all productivity levels amount to approximately $8,500.\(^{14}\) Finally, we use the ratio of exogenous budget expenditures \( E \) to aggregate production of 0.25 as in Saez (2001).

### 6.3 Results for nondifferentiated taxation

Figure 4 illustrates the simulation outcomes for potential earnings up to $500,000. In the no-migration case (dashed line in Figure 4A), the government chooses a higher marginal tax rate than in the migration case (solid lines), in line with Proposition 2. For low income levels, there is no difference in the marginal tax rate schedules. Migration-induced productivity increases are small in this income range, and tax liabilities are rather low as well, so that the migration effect is negligible. However, this is no longer true for higher levels of income, on account

\(^{14}\)Our analysis considers migration gains on a yearly basis. Interpreting migration costs as a perpetually accruing disutility and applying a reasonable discount rate, this cost level corresponds to the range of migration-cost estimates obtained by Kennan and Walker (2011) or Bayer and Juessen (2012).
of the higher productivity gains and progressive taxation.

Moreover, differences in the regional productivity distributions also matter for the difference between the two schedules, since they also determine the size of the migration effect and the corresponding fiscal externality. As Figure 4A (baseline case) shows, the migration effect reduces marginal tax rates by up to six percentage points in the higher income range.

In Figure 4B we plot the marginal tax rate differences between the migration and the no-migration case for alternative values of the migration elasticity. This illustrates that the relationship between the tax rate reduction due to productivity-enhancing migration relative to the no migration case is quite sensitive to the migration elasticity, and that, overall, this relationship is not monotone. Consistently with the theory, the differences in marginal tax rates are reduced for sufficiently low values of $\eta$, as can be seen from the comparison of the tax rate differences for $\eta = 0.2$ and $\eta = 1$ in 4B, respectively.\footnote{Trivially, the differences in marginal tax rates completely disappear for $\eta \to 0$.} Note also from 4B that the marginal tax rate differences may be substantially more pronounced than the six percentage points in our baseline case, exceeding twelve percentage points for $\eta = 1$ for some ability levels.

Finally, further simulations show that, as usual, a higher labor supply elasticity lowers marginal tax rates throughout the ability distribution, and that this also reduces the gap between the migration and the no-migration tax schedule (not shown). Intuitively, lower progressivity tends to reduce the interregional difference in tax payments between individuals of the same innate productivity, so that the fiscal migration externality is reduced.

\section*{6.4 Results for differentiated taxation}

The results for differentiated taxation are depicted in Figures 5 and 6. We use the same baseline parameter settings for migration costs and the exogenous government expenditure requirement $E$ as before. Figure 5A illustrates the case of differentiated taxation without migration (fixed-residence case). This is a useful
benchmark, since it corresponds to the standard tagging case with an exogenous tag. The marginal tax rates in the high-productivity region are very similar to those in the low-productivity region for low levels of potential income. However, in the range of potential incomes between approximately $85,000 and $350,000 the mass of individuals in region B is substantially larger than in region A, so that the larger potential for behavioral adjustments reduces optimal marginal tax rates in the more productive region below the level of marginal tax rates in the low productivity region. We compare this fixed residence benchmark with the other three graphs of Figure 5, where we allow for productivity-increasing migration and display the optimal differentiated marginal tax rate schedules for different values

Figure 5: Optimal differentiated tax simulations for the US based on data from PSID 2008/09 and TAXSIM.
of the migration elasticity.

Figure 5B is based on a migration elasticity $\eta = 1.5$. In comparison to the fixed-residence case, marginal tax rates are higher for intermediate abilities and for individuals with high potential earnings in region A. The opposite is true in region B, where marginal tax rates are substantially reduced. Additionally, we display the optimal marginal tax rates for a lower and a higher migration elasticity, $\eta = 2$ and $\eta = 0.5$, respectively, in Figure 5C and 5D. In all cases U-shaped marginal tax rate patterns reappear for both regions, but the regional tax rates are substantially different. For all analyzed values of the migration elasticity, regional marginal tax rates are relatively closely together at low potential income levels, but they begin to diverge sharply for potential earnings above $50,000$. Above this level, marginal tax rates in the less productive region are substantially higher than those in the more productive region, and only converge again for very high levels of potential income. This differentiation illustrates the downward pressure of the migration effect on marginal tax rates in the high-productivity region and the reverse pressure in the low-productivity region as suggested by Proposition 4. Thus, only for very low or high abilities do the marginal tax rates in region B approximate those in region A, whereas for a very substantial intermediate income range lower tax rates in region B are optimal.

With a lower migration elasticity of $\eta = 0.5$, the difference in marginal tax rates is even more pronounced and increases to more than 20 percentage points (Figure 5D) whereas tax rate convergence occurs only at substantially higher ability levels (not shown). For a higher migration elasticity, as displayed in Figure 5C, the regional marginal tax rate spread is reduced, but still larger than in the fixed residence case. To sum up the comparison to the fixed residence case, productivity-enhancing migration substantially alters the shape of optimal differentiated taxation relative to the benchmark of a purely exogenous tag of fixed productivity differences.

Figure 6 uses innate earnings – defined as potential income with innate productivity, which corresponds to potential income in the low-productivity region.
Figure 6: Further simulation results for the migration baseline case of differentiated taxation, based on PSID and TAXSIM. Parameter values for all illustrations: $\varepsilon = 0.25$, $\eta = 1.5$, $\gamma = 1$.

– to compare the optimal tax treatment of individuals of the same innate ability in the two regions. This takes into account that individuals will have a different realized productivity and correspondingly a different potential income in the high-productivity region. We depict optimal absolute tax liabilities in Figure 6A and gross incomes in Figure 6B. We observe a basic income support of roughly $15,000 ($17,000) in region B (A), which is taxed away quickly in both regions as earnings increase. Gross incomes begin to considerably diverge from about $50,000 innate earnings and show a roughly constant surplus in the metropolitan region that occurs around $250,000 innate earnings.

Figure 6C shows the same simulation results as Figure 5B, but considers the
optimal marginal tax rate schedules as functions of innate earnings. This allows us to compare the optimal marginal tax rates an individual of a given innate productivity should face in both regions. In addition, we show the ratio of tax liability differences to realized income differences between the two regions, which is the implicit migration tax rate. The implicit migration tax is greater than one for low ability levels, but falls quickly over the range of $30,000–$100,000 in innate earnings. It is roughly constant at about 44% above these levels of innate earnings. This corresponds to the evidence from Figure 6A and B, which indicate almost constant differences of the tax liability schedules and the gross incomes for high earnings levels.

Finally, we display the absolute migration gain net of taxes (net gain) in Figure 6D. The displayed pattern corresponds to the lag function $\kappa(n)$ (Figure 3B), but additionally takes the effects of differentiated taxation into account. It is evident that, except for the lowest innate earnings levels, there is a positive and increasing net gain from migration, until the constant level of $\kappa$ is reached.

It is interesting to compare our optimally differentiated tax schedule with the actual regional differentiation of income taxation due to cost-of-living differences. Albouy (2009) argues that undifferentiated federal income taxation taxes inhabitants of urban agglomerations effectively more heavily than those of rural areas, resulting in a spatial distortion of economic activity. Our analysis of optimal redistribution with location-dependent productivity suggests that optimally differentiated taxation should bear more heavily on low-productivity regions, with the exception of the very low and very high income earners. Thus, the de facto differentiation in favor of low-productivity regions appears to be the opposite of an optimally differentiated tax system. Implicit regional tax differentiation may therefore additionally be criticized from a redistribution perspective with location-specific productivity.
7 Concluding remarks

Regional inequality and the corresponding possibility of productivity-enhancing migration can be an important determinant of the optimal redistributive tax-transfer scheme. The migration possibility implies a fiscal externality that modifies the equity-efficiency trade-off beyond the intensive labor supply margin, and our simulations indicate that this additional constraint to redistribution can be quantitatively important. However, our analysis abstracts from a number of potentially important aspects.

First, we restrict the central government to the use of a unified or regionally differentiated tax scheme; it is not allowed to use targeted subsidies to migrants only. If such targeted transfers were available to the government, they could potentially loosen the trade-off between redistribution and internal migration. However, a fixed migration subsidy typically cannot eliminate the problem completely, since the fiscal migration externality differs by earnings level. Even if the migration subsidy could be adjusted by earnings level, the adjustments would have to be incentive-compatible, implying an additional constraint for tax policy.

Secondly, regional productivity differences are partly reflected in local prices of nontradable goods, rents, and house prices, which also reduces migration incentives. While this is important, it does not challenge our intuition. The value of an additional dollar of tax revenue for the government is independent of the region where this dollar is raised. Thus, the fiscal migration externality also exists with regional cost of living differences and should be taken into account.

Finally, we have used a static framework. One may argue that migration also contains an inherently dynamic aspect. At the same time, to the extent that the migration costs are recurring costs, say, because the disutility of being in a less preferred region accrues every period, our approach easily maps into a dynamic framework. However, further research may explore how the potentially stochastic evolution of regional productivity differences and the option of repeated migration may qualify our results.

Our results have immediate policy implications. Tax policy should not only
worry about external migration, but also needs to consider the potential role of productivity-increasing internal migration for a progressive tax-transfer schedule. This constraint is less important for countries that are characterized by low regional inequality. However, in countries where regional inequality is substantial, policy makers should carefully assess how tax progressivity may hurt productivity-enhancing interregional migration and design their tax-transfer schemes accordingly.

8 Appendix A: derivation of the optimal tax formulae

We now show that the optimal tax formulae (6) and (8) and (9) can also be rigorously derived by standard optimal control techniques. The equivalence of the expression in terms of \( n \) and \( z \) is shown in Supplement D. We start with the differentiated case, since the unified case can be interpreted as the same problem with the additional constraint of the tax schedules to be identical in both regions.

8.1 Regionally differentiated taxation

The government maximizes

\[
W = \int_{n_{\text{min}}}^{n_{\text{max}}} \left[ \int_{q_{B}}^{+\infty} \Psi \left( V_B(\omega(n)) + q^h \right) p(q|n) dq + \int_{0}^{q_{A}} \Psi \left( V_B(\omega(n)) - q^c \right) p(q|n) dq \right. \\
+ \int_{q_{A}}^{+\infty} \Psi \left( V_A(n) + q^h \right) p(q|n) dq + \int_{0}^{q_{B}} \Psi \left( V_A(n) - q^c \right) p(q|n) dq f(n) dn,
\]

where \( q_A(n) = \max \{ V_B(\omega(n)) - V_A(n), 0 \} \), \( q_B(n) = \max \{ V_A(n) - V_B(\omega(n)), 0 \} \), \( q = q^c + q^h \), and either \( q^c = 0 \) or \( q^h = 0 \). The first term in this expression stands for the social welfare from the population of region B who did not move, the second term stands for that of the population moved from A to B, the third term is for those who stayed in A, and the fourth term is for those who moved from B to A. Note that either the second or the fourth term is equal to zero, because migration in both direction at the same ability level is not possible. The maximization is subject to

\[
\int_{n_{\text{min}}}^{n_{\text{max}}} \left[ \left( z_B - \omega(n) h \left( \frac{z_B}{\omega(n)} \right) - V_B(\omega(n)) \right) (1 + P(q_A|n) - P(\bar{q}_A|n)) \\
+ \left( z_A - nh \left( \frac{z_A}{n} \right) - V_A \right) (1 + P(\bar{q}_B|n) - P(\bar{q}_A|n)) \right] f(n) dn \geq E
\]

\[\text{16}\] We drop the arguments of \( \bar{q}_A \) and \( \bar{q}_B \) for the ease of notation.
and the corresponding incentive compatibility constraints. Note that either $P(q_A|n)$ or $P(q_B|n)$ is zero for the same reason as discussed above.

Let the Hamiltonian be $H(z_A, z_B, V_A, V_B, \lambda, \mu_A, \mu_B, n)$. The necessary conditions are:

1. There exist absolutely continuous multipliers $\mu_A(n), \mu_B(n)$ such that on $(n_{\min}, n_{\max})$
   \[
   \hat{\mu}_B(n) = -\frac{\partial H(n)}{\partial V_B(n)}, \quad \hat{\mu}_A(n) = -\frac{\partial H(n)}{\partial V_A(n)}
   \]
   almost everywhere with $\mu_i(n_{\min}) = \mu_i(n_{\max}) = 0$.

2. We have $H(z_i(n), V_i, \lambda, \mu_i, n) > H(z_i, V_i, \lambda, \mu_i, n)$ almost everywhere in $n$ for all $i$.

The first order conditions are $\frac{\partial H}{\partial x_{A}} = 0$, $\frac{\partial H}{\partial x_{B}} = 0$.

Uniqueness of $z_A$ and $z_B$ that solve the equations above can be established in a similar way to Kleven et al. (2009), using the assumption that $\varphi(x) = (1 - h'(x)) / (xh''(x))$ is decreasing in $x$. The FOCs for Region A and B are presented in Supplement A, where we also lead the reader through the derivation steps and provide additional representations of the optimal tax formulae:

\[
\begin{align*}
\frac{\tau_A}{1 - \tau_A} &= \frac{1}{nf(n)\varepsilon_A (1 + P(q_B|n) - P(q_A|n))} \int_{n_{\min}}^{n_{\max}} \left[ (1 - g_A(n')) (1 + P(q_B|n') - P(q_A|n')) 
   
   - (T_A - T_B) \left( p(q_B|n') + p(q_A|n') \right) \right] f(n') \, dn', \\
\frac{\tau_B}{1 - \tau_B} &= \frac{1}{\omega(n)f(n)\varepsilon_B (1 + P(q_A|n) - P(q_B|n))} \int_{n_{\min}}^{n_{\max}} \left[ (1 - g_B(n')) (1 + P(q_A|n') - P(q_B|n')) 
   
   - (T_B - T_A) \left( p(q_B|n') + p(q_A|n') \right) \right] f(n') \, dn',
\end{align*}
\]

for the marginal rates in A and B, respectively. The formulae are similar to Kleven et al (2009) except that two terms (rather than one) reflect the possibility of either immigration to or emigration from the given region. Note that for each $n$, there are two mutually exclusive scenarios: either there is migration from A to B (and $V_B(\omega(n)) > V_A(n)$).

### 8.2 Delayed optimal control

For the next section, we will need some results in delayed optimal control theory. In particular, we adapt Göllmann et al. (2008) setting that adresses delayed arguments of constant size. We consider variable “delays” of the size $\omega(n) - n$ in both the state variable $x(n)$ (in our case this is $V(n)$) and in the control variable $u(n)$ (in our case this is $z(n)$). Consider a welfare maximization problem. Because social welfare depends on the indirect utilities only, our objective function will not depend on control variables.

We have the following retarded optimal control problem (ROCP):

\[
J(u, x) = \int_{n}^{b} L(n, x(n), x(\omega(n))) \, dn,
\]

subject to the differential equation (ICC in our context, can be formulated without delayed variables and without states on the rhs) and inequality constraint (government
\[ \dot{x}(n) = f(n, u(n)), \quad n \in [a, b] \quad (11a) \]
\[ \int_a^b C(n, u(n), u(\omega(n)), x(n), x(\omega(n))) \, dn \geq 0. \quad (11b) \]

For convenience, the functions
\[ L : [a, b] \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R} \]
\[ f : [a, b] \times \mathbb{R}^n \to \mathbb{R} \]
\[ C : [a, b] \times \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R} \]

are assumed to be twice continuously differentiable wrt all arguments. A pair of functions \((u, x)\) is called an admissible pair for the problem (ROCP) if the state \(x\) and control \(u\) satisfy (11a) and (11b). An admissible pair \((\hat{u}, \hat{x})\) is called locally optimal pair or weak maximum for (ROCP), if
\[ J(u, x) \leq J(\hat{u}, \hat{x}) \]
hold for all \((u, x)\) admissible in a neighbourhood of \((\hat{u}, \hat{x})\). Define the Hamiltonian for (ROCP) as
\[ H(n, x, x^w, u, u^w, \mu, \lambda) := L(n, x, x^w) + \lambda C(n, u, u^w, x, x^w) + \mu f(n, u) . \]

Göllmann et al (2008) show that a necessary condition for \((u, x)\) to be locally optimal is existence of a costate absolutely continuous function \(\mu : [a, b] \to R\), a multiplier function \(\lambda : [a, b] \to R\) such that the following conditions hold for all \(t \in [a, b]\):

(i) adjoint differential equation
\[
\dot{\mu}(n) = -\frac{\partial H(n)}{\partial x} - \left[\omega(n) \omega(b)\right] \frac{\partial H(\omega^{-1}(n))}{\partial x^w} \]
\[= -\frac{\partial H(n, x(n), x(\omega(n)), u(n), u(\omega(n)), \mu(n), \lambda(n))}{\partial x} \]
\[ - \left[\omega(n) \omega(b)\right] \frac{\partial H(n, x(\omega^{-1}(n)), x(n), u(\omega^{-1}(n)), u(n), \mu(\omega^{-1}(n)), \lambda(\omega^{-1}(n)))}{\partial x^w} , \]

(ii) transversality condition
\[ \mu(a) = \mu(b) = 0, \]

(iii) local maximum condition
\[ \frac{\partial H(n)}{\partial u} + I[\omega(n) \omega(b)] \frac{\partial H(\omega^{-1}(n))}{\partial u^w} = 0, \]

(iv) nonnegativity of multiplier and complementarity slackness.
8.3 Non-differentiated tax schedule

In this case the tax schedules in two regions must be identical, and hence the indirect utilities also are (there are no differences in preferences). The government problem is to maximize

\[
W = \int_{n_{\min}}^{n_{\max}} \left[ \int_0^{+\infty} \Psi \left( V(\omega(n)) + q^B \right) p(q|n) dq + \int_0^{\bar{q}} \Psi \left( V(\omega(n)) - q^C \right) p(q|n) dq \right. \\
\left. + \int_{\bar{q}}^{+\infty} \Psi \left( V(n) + q^B \right) p(q|n) dq \right] f(n) dn,
\]

where \( \bar{q} = V(\omega(n)) - V(n) \), and either \( q^B \) or \( q^C \) is equal to zero. We have also dropped the subscript B from the omega function for more parsimonious notation. The maximization is subject to

\[
\int_{n_{\min}}^{n_{\max}} \left[ \left( z(\omega(n)) - \omega(n)h \left( \frac{z(\omega(n))}{\omega(n)} \right) - V(\omega(n)) \right) (1 + P(\bar{q}|n)) \right. \\
\left. + \left( z - nh \left( \frac{z}{n} \right) - V \right) (1 - P(\bar{q}|n)) \right] f(n) dn \geq E,
\]

where the superscript \( w \) stands for the individuals with productivity \( \omega(n) \). Note that in the uniform case there cannot be migration from B to A, as this would imply \( V(\omega(n)) < V(n) \) that contradicts incentive compatibility (the productivity type \( \omega(n) \) can pretend to have productivity \( n \) without any costs).

Let the Hamiltonian be \( H(z, z^w, V, V^w, \lambda, \mu, n) \). This is a delayed optimal control problem analogous to the one formally analyzed by Göllmann et al. (2008) in its entire generality. The difference is that whereas Göllmann et al. have a lag of fixed size over the whole domain of their functions, our lag is a smooth increasing function of \( n \), namely \( \omega(n) - n \). The necessary conditions for optimal control in such a setting is presented in Abdeljawad et al (2009). Namely, in our context the necessary conditions for the maximum are:

1. There exist absolutely continuous multipliers \( \mu(n) \) such that on \( (n_{\min}, n_{\max}) \)
   \[
   \frac{\partial H(n)}{\partial V(n)} - 1_{[\omega(n_{\min}), \omega(n_{\max})]} \frac{\partial H(\omega^{-1}(n))}{\partial \omega^{-1}(n)} \frac{\partial H(\omega^{-1}(n))}{\partial z^n_B} = 0
   \]
   almost everywhere with \( \mu(n_{\min}) = \mu(n_{\max}) = 0 \).

2. We have \( H(z(n), z^w(n), V, V^w, \lambda, \mu, n) > H(z, z^w, V, V^w, \lambda, \mu, n) \) almost everywhere in \( n \) for all \( z \). The first order condition is
   \[
   \frac{\partial H}{\partial z} + I_{[\omega(n_{\min}), \omega(n_{\max})]} \frac{\partial H(\omega^{-1}(n))}{\partial z^n_B} = 0
   \]

The fact that this condition describes a global maximum can be established similar to Kleven et al. (2009), using the assumption that \( \varphi(x) = (1 - h'(x)) / (x h''(x)) \) is decreasing in \( x \).

\[
\frac{\mu z}{n} h'' \left( \frac{z}{n} \right) + \lambda \left( 1 - h' \left( \frac{z}{n} \right) \right) \left( (1 - P(\bar{q}|n)) f(n) + (1 + P(\bar{q}_1 | \omega^{-1}(n))) f(\omega^{-1}(n)) \right) = 0
\]

\[
\varphi \left( \frac{z}{n} \right) = - \frac{\mu(n)}{\lambda n (1 - P(\bar{q}|n)) f(n) + (1 + P(\bar{q}_1 | \omega^{-1}(n))) f(\omega^{-1}(n))}
\]

LHS is decreasing in \( z_i/n \) whereas RHS is constant, which implies that \( z_i(n) \) is a unique
solution and a global maximum indeed. Continuity can be then established in a way similar to Kleven et al (2009). Further derivation is presented in the Supplement A, whereby we arrive at

\[ \int_{\omega^{-1}(n)}^{n_{\text{max}}} (1 - g_B(n')) \left( 1 + P(\bar{q}|n') \right) f(n') \, dn' + \int_{n}^{n_{\text{max}}} (1 - g_A(n')) \left( 1 - P(\bar{q}|n') \right) f(n') \, dn' - \int_{\omega^{-1}(n)}^{n} (T(\omega(n')) - T(n')) \, p(\bar{q}|n') f(n') \, dn' = \frac{\tau}{1-\tau} \times \]

\[ \times n \bar{\varepsilon} \left( (1 - P(\bar{q}|n)) f(n) + (1 + P(\bar{q}|\omega^{-1}(n))) f(\omega^{-1}(n)) \right). \]  

(12)

Additional interpretation of this formula is also left to the Supplement A.

9 Appendix AA: Further propositions and proofs

9.1 Proof of Proposition 2

Proof. Going through the derivation of the optimal tax formula in the Appendix A under the assumption that the effect of tax on migration decision is neglected, i.e. \( \frac{\partial g}{\partial z} = \frac{\partial g}{\partial V} = 0 \), we arrive at the optimal tax formula (12) short of the term

\[ \int_{\omega^{-1}(n)}^{n} (T(\omega(n')) - T(n')) \, p(\bar{q}|n') f(n') \, dn'. \]

If this is non-positive (that is equivalent to \( D(z) \leq 0 \)), the result immediately follows. If marginal tax rates are positive, this condition is always fulfilled. ■

9.2 Proof of Proposition 5

Proof. Under the assumptions formulated in the text, \( V_A(n) \) and \( V_B(n) \) are increasing in \( n \) without bound, because \( \tau_A^\infty < 1, \tau_B^\infty < 1 \). As \( \Psi' > 0 \) is decreasing, it converges to some \( \bar{\psi} \geq 0 \). Then, we have

\[ g_A(n) = \frac{\int_{\bar{q}_A}^{+\infty} \Psi'(V_A(n) + q^h) \, p(q|n') \, dq + \int_{0}^{\bar{q}_B} \Psi'(V_A(n) - q^c) \, p(q|n) \, dq}{\lambda \left( 1 + P(\bar{q}_B|n) - P(\bar{q}_A|n) \right)}, \]

\[ g_B(n) = \frac{\int_{\bar{q}_B}^{+\infty} \Psi'(V_B(\omega(n')) + q^h) \, p(q|n') \, dq + \int_{0}^{\bar{q}_A} \Psi'(V_B(\omega(n)) - q^c) \, p(q|n) \, dq}{\lambda \left( 1 + P(\bar{q}_A|n) - P(\bar{q}_B|n) \right)}, \]

which converge to

\[ g_A^\infty = g_B^\infty = \frac{\bar{\psi}}{\lambda}. \]

If \( T_B - T_A \) converges, it must be that \( \tau_A^\infty = \tau_B^\infty = \tau^\infty \). But since

\[ h' \left( \frac{z_i}{n_i} \right) = 1 - \tau_i (z_i), \]
\( z_i / n_i \) converges and hence elasticities converge to the same limit \( \tilde{\varepsilon} \). Moreover, \( \lim_{n \to \infty} \frac{zA}{n} = \lim_{n \to \infty} \frac{zB}{\omega(n)} \). Because \( P(q|n) \) and \( qA, qB \) converge, \( P(q|n) \) and \( qA, qB \) converge to \( P^\infty(q_i^\infty) \) and \( P^\infty(q_i^\infty) \). The Pareto distribution implies that \( (1 - F(n)) / (nf(n)) = 1/a \) in the tail. Take the limit of our optimal tax formulae to get

\[
\frac{1}{a \varepsilon}[1 - \frac{\chi}{\lambda} + \frac{\Delta T^\infty (p^\infty(qB) + p^\infty(qA))}{1 + P^\infty(qB) - P^\infty(qA)}] = \frac{\tau^\infty}{1 - \tau^\infty},
\]

\[
\frac{1}{a \varepsilon}[1 - \frac{\chi}{\lambda} - \frac{\Delta T^\infty (p^\infty(qB) + p^\infty(qA))}{1 + P^\infty(qB) - P^\infty(qA)}] = \frac{\tau^\infty}{1 - \tau^\infty},
\]

for the marginal rates in Region A and B, respectively. The right hand sides are equal, so we need \( \Delta T^\infty = 0 \) for the left hand sides to be equal as well.\(^{17} \)

### 9.3 Proof of Proposition 6

**Proof.** The maximization problem of the government with the restriction that \( \Delta T = C \) is constant in \( n \) is

\[
W = \int_{n_{\text{min}}}^{n_{\text{max}}} \left[ \int_0^{+\infty} \Psi \left( V(\omega(n)) + q^h \right) p(q|n)dq + \int_{\bar{q}}^{+\infty} \Psi \left( V(\omega(n)) - q^c \right) p(q|n)dq 
+ \int_{\bar{q}}^{+\infty} \Psi \left( V(n) + q^h + C \right) p(q|n)dq \right] f(n)dn,
\]

where \( \bar{q} = V(\omega(n)) - V(n) - C \), and either \( q^h \) or \( q^c \) is equal to zero, and we assume \( C \) is small enough not to induce “reverse” migration (to the low productivity region). The maximization is subject to

\[
\int_{n_{\text{min}}}^{n_{\text{max}}} \left[ \left( z(\omega(n)) - \omega(n)h \left( \frac{z(\omega(n))}{\omega(n)} \right) - V(\omega(n)) \right) (1 + P(q|n)) 
+ \left( z - nh \left( \frac{z}{n} \right) - V - C \right) (1 - P(q|n)) \right] f(n)dn \geq E.
\]

Note that we express everything here in terms of Region B taxes - that is why \( C \) appears in the expressions for Region A as a correction term to increase indirect utility (in the objective function) or to reduce the tax revenue (in the government budget constraint). By the envelope theorem,

\[
\frac{\partial W^*}{\partial C} = \int_{n_{\text{min}}}^{n_{\text{max}}} \left[ \Psi \left( V(n) + q^h + C \right) - \Psi \left( V(\omega(n)) - \bar{q}^c \right) \right] p(q|n) 
+ \int_{\bar{q}}^{+\infty} \Psi \left( V(n) + q^h \right) p(q|n)dq - \lambda (1 - P(q|n)) 
- \lambda (T(\omega(n)) - T(n) + C) p(q|n)f(n)dn.
\]

\(^{17}\)Note that the optimal tax formula for the uniform case simplifies to

\[
2\varepsilon^\infty \frac{\tau^\infty}{1 - \tau^\infty} = \frac{1 - F(n)}{nf(n)} \left( 2 - \frac{\psi}{\lambda} \right),
\]

which is identical to (10) under the Pareto distribution and proper rescaling of the Lagrange multiplier.
\[
\frac{\partial W^*}{\partial C} \big|_{C=0} = \int_{q_{\text{min}}}^{q_{\text{max}}} \int_{q_{\text{min}}}^{+\infty} \left( \Psi'(V(n) + q^h) - \lambda \right) p(q|n) dq - \lambda (T(\omega(n)) - T(n)) p(\bar{q}|n) f(n) dn,
\]
which is negative, if \( \Psi'(V(n) + q^h) / \lambda = g_A \leq 1 \) and \( T(\omega(n)) > T(n) \) (a sufficient condition is that the marginal tax rate is positive everywhere).

## 9.4 Proof of Proposition 7

**Proof.** A separable tax schedule implies that \( T_B - T_A \) is constant. Since \( z_A/n = z_B/\omega(n) \), we have
\[
\dot{q}_A = V_B - V_A = (\omega(n) - n) \left( \frac{z}{n} - h \left( \frac{z}{n} \right) \right) - (T_B - T_A),
\]
so we can write
\[
\dot{q}_A = (\omega'(n) - 1) \left( \frac{z}{n} - h \left( \frac{z}{n} \right) \right) - (T_B' - T_A').
\]
In particular, under separable taxation and \( \omega'(n) = 1 \), we have \( \dot{q}_A = 0 \). At that point, for the cost-of-moving model
\[
\frac{d(g_A - g_B)}{dn} = \left[ \frac{\Psi''(V_A(n))}{\lambda} - \frac{\Psi''(V_A + \bar{q}_A) + \int_0^{\bar{q}_A} \Psi'\left(V_A + \bar{q}_A - q^h\right) p(q) dq}{\lambda (1 + P(\bar{q}_A))} \right] V_A < 0
\]
iff \( \Psi' \) is convex.

Similar to Kleven et al. (2006, 2009) we can consider a tax reform introducing a little bit of “negative jointness” (a lower marginal tax for the higher productivity region). This reform has two components. Above ability level \( n \), we increase the tax in Region A and decrease the tax in Region B. Below ability level \( n \), we decrease the tax in Region A and increase the tax in Region B. These tax burden changes are associated with changes in the marginal tax rates on earners around \( n \). The direct welfare effect created by redistribution across regions at each income level:
\[
dW = \frac{dT}{F(n)} \int_{n_{\text{min}}}^{n} (g_A(n') - g_B(n')) f(n') dn'
- \frac{dT}{1 - F(n)} \int_{n}^{n_{\text{max}}} (g_A(n') - g_B(n')) f(n') dn'.
\]
Because \( g_A - g_B \) is decreasing, \( dW > 0 \). Second, there are fiscal effects associated with earnings responses induced by the changes in \( \tau_A \) and \( \tau_B \) around \( n \). Since the reform increases the marginal tax rate in Region A around \( n \) and reduces it in Region B, the earnings responses are opposite. As we start from \( \tau_A = \tau_B \), and hence identical elasticities, \( \varepsilon_A = \varepsilon_B \), the fiscal effects of earning responses cancel out exactly. Finally, the reform creates migration responses. Above \( n \), migration to B will be induced. Below \( n \), migration to B will be inhibited. The fiscal implications of these responses cancel out exactly only if \( \omega'(n) = 1 \). The elasticity \( \eta \) is constant in this case and since initially \( T_A - T_B \) is constant, the revenue gain from migrants above \( n \) will be compensated by the revenue loss from migrants below \( n \). By the same logic, for \( \omega'(n) > 1 \) the gain from migration will be stronger then the loss from it, so we will have another positive effect. With \( \omega'(n) < 1 \) the revenue gain from migration is smaller than the loss, so a bit of negative jointness is not necessarily optimal. To complete the proof, our reasoning needs to hold for \( \omega'(n) > 1 \), i.e. \( \dot{g}_A - \dot{g}_B < 0 \) also for this case. Differentiating \( g_A - g_B \) in this
case, we have
\[ \hat{g}_A - \hat{g}_B = \hat{V}_A \frac{\Psi''(V_A)}{\lambda} - \omega(n) \hat{V}_A \frac{\Psi''(V_A + \bar{q})}{\lambda (1 + P(\bar{q}))} - \frac{g_A - g_B}{1 + P(\bar{q})} p(\bar{q}) (\omega'(n) - 1) \hat{V}_A < 0, \]

The first two terms are negative, because \( \Psi' \) is convex by assumption. The third term is negative, because \( g_A > g_B \) (which follows from concavity of \( \Psi \)).

9.5 Two propositions using the alternative benchmark of the ex ante productivity distribution

**Proposition 8** In a country with regional productivity differences and internal migration, the optimal non-differentiated marginal tax rates may be higher or lower relative to a benchmark without internal migration and the ex ante productivity distribution. Assuming exogenous marginal welfare weights, they are lower if

\[
\int_{\omega^{-1}(n)}^{n} (1 - g_B(n')) P(\bar{q}|n') f(n') \, dn' - \int_{\omega^{-1}(n)}^{n} (T(\omega(n')) - T(n')) p(\bar{q}|n') f(n') \, dn' < 0.
\]

**Proof.** The optimal tax formula in case of unified taxation is presented by (12). For a government that does not take into account the possibility of migration, optimal marginal tax rates are implicitly defined by

\[
\int_{\omega^{-1}(n)}^{n} [2 - g_A(n')] f(n') \, dn' + \int_{\omega^{-1}(n)}^{n} (1 - g_B(n')) f(n') \, dn' = \frac{\tau}{1 - \tau} (f(n) + f(\omega^{-1}(n))).
\]

Comparing the two expressions, we arrive at the condition (15).

**Proposition 9** Starting from two identical regions, introducing a marginal difference in productivity distribution lowers optimal marginal tax if and only if

\[
\frac{1 - g_B(n)}{2} + \frac{\int_{n}^{\max} [2 - g_A(n') - g_B(n')]}{4 (f(n))^2} \left( \frac{P(\bar{q}|n)}{\partial n} f(n) + f'(n) \right) < 0.
\]

**Proof.** We express the marginal tax rate \( \frac{\hat{\tau}}{1 - \hat{\tau}} \) from the optimal tax formula (12) under the assumption that \( \omega(n) = n + \Delta, \) take a derivative of it with respect to \( \Delta, \) and evaluate it at \( \Delta = 0, \) keeping in mind that there is no migration at this point. The
resulting expression is proportional to the left hand side of (16). Correspondingly, the marginal tax rate decreases with the introduction of marginal productivity differences if this expression is negative and it increases in case it is positive.

We can see that the terms in (16) related to the revenue effect are always positive. Thus, a sufficient condition for an increase in marginal tax is that

\[
P(q|n) \frac{\partial}{\partial n} f(n) + f'(n) \geq 0.
\]

This is satisfied for independent distribution of costs \( P(q|n) = 0 \) and a uniform distribution of ability. On the other hand, if the distribution of ability is sufficiently “decreasing”, like the Pareto distribution, for example, then introducing marginal productivity differences puts downward pressure on marginal taxes.

For a linear approximation, \( T''(z) = 0 \), so we get \( n\hat{z}(n) = z(n) \) indeed. This completes the proof of equivalence for the case of non-differentiated taxation. The derivation for differentiated taxation is analogous.
REFERENCES


KRUEGER, D., and A. LUDWIG (2013): "Optimal Progressive Taxation and Education


PANEL STUDY OF INCOME DYNAMICS (2013): Public use dataset. Produced and distributed by the Institute for Social Research, University of Michigan, Ann Arbor, MI (2013)


