Public Economics — The Macroeconomic Perspective
Chapter 5: Income Taxation

University of Augsburg

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6 The Real Business Cycle Model and Stochastic Taxes
1. The welfare costs that are associated with the taxation of labor income are substantial in both partial and general equilibrium.

2. Seminal result from optimal taxation: capital income should not be taxed in the long run.

3. We estimate the US Laffer curve.

4. The US government can still raise its revenues from labor and capital income taxation by approximately 10% of GDP.

5. We derive growth-maximizing taxes on total income and on capital income.
Contents and Results

6 The quantitative effects of higher taxes on economic growth are derived in a Dynamic General Equilibrium (DGE) model and are shown to be substantially higher than those typically found in growth regressions.

7 Stochastic taxes improve the time series properties of the real business cycle (RBC) model with respect to the volatility of aggregate demand components and the dynamics of labor and wages.
Empirical Regularities

Empirical Observations

1. Tax revenues vary considerably across OECD countries.
2. The United States has among the lowest shares of revenue in GDP, amounting to 26.2% in 2015.
3. The Scandinavian countries of Denmark and Finland together with France have among the highest taxes in the OECD, amounting to 45.9%, 43.9%, and 45.2% of GDP, respectively.
4. Total tax revenue are upward sloping for the Eurozone countries France, Italy, and Spain (and, to a smaller extent, Germany) as well as Japan, while there is no discernable trend in tax revenue (as % of GDP) in the UK and US.
Tax revenue relative to GDP in OECD countries 2015
Tax revenue (% of GDP), 1965-2016

- Canada
- France
- Germany
- Italy
- Japan
- Spain
- UK
- US

Tax revenue (% of GDP) over time from 1965 to 2016 for various countries.
### US tax composition in 2015 (% of GDP)

<table>
<thead>
<tr>
<th>Tax</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total taxes</td>
<td>26.22</td>
</tr>
<tr>
<td>Tax on personal income</td>
<td>10.63</td>
</tr>
<tr>
<td>Tax on corporate profits</td>
<td>2.23</td>
</tr>
<tr>
<td>Social Security contributions</td>
<td>6.20</td>
</tr>
<tr>
<td>Tax on payroll</td>
<td>0.00</td>
</tr>
<tr>
<td>Tax on property</td>
<td>2.70</td>
</tr>
<tr>
<td>Tax on goods and services</td>
<td>4.46</td>
</tr>
</tbody>
</table>

**Notes:** Data are retrieved from the OECD. See Appendix 5.2 for a description of the data source.
Marginal and effective US income tax rates, 2016
Income Tax schedules

Empirical Observations on Income Tax Schedules

1. In the US, only 4.2% of households earned an income in excess of $200,000 in 2010 according to US census data, meaning that most taxpayers face a marginal tax rate below 28.0%.

2. The German income tax system is much more progressive than the US income tax system.

3. While the average and marginal income tax rates amount to 17% and 25% for an income of $100,000 in the US, the corresponding income tax rates for an income of €90,000 amount to approximately 24% and 37% in Germany.

4. For a household income equal to €107,332, a German couple had to pay a marginal tax rate equal to 44.3% in 2016.
Average income tax rate, Germany (married household)
Marginal income tax rate, Germany (married household)
Income Tax schedules: Progressivity measures

Progressivity Measures

1. Yield elasticity

\[ \eta_{T,Y} = \frac{dT(Y)}{dY} \frac{Y}{T(Y)} = \frac{\tau'}{\bar{\tau}} \]  

where we speak of a regressive, proportional or regressive tax system if:

\[ \eta_{T,Y} = \begin{cases} < 1 & : \text{regressive tax code} \\ = 1 & : \text{proportional} \\ > 1 & : \text{progressive} \end{cases} \]
Income Tax schedules: Progressivity measures

Residual elasticity:

$$\eta_{Y^n,Y} = \frac{dY^n}{dY} \frac{Y}{Y^n} = \frac{1 - \tau'}{1 - \bar{\tau}}.$$ 

residual = net income after taxes \( Y^n = Y - T(Y) \)
US income tax rates, 1948-2008
Observations on Labor and Capital Income Tax Rates in the US

- Capital income was taxed more heavily than labor income in the last century: The average capital and labor income tax rates amount to 41% and 23%.

- In recent years, the tax rates on capital and labor income have converged in the US.

- Both the cyclical components of the capital and the labor income tax rates $\tau^L$ and $\tau^K$ are positively correlated with output.
Observations on Labor and Capital Income Tax Rates in the US

Both tax rates $\tau^L$ and $\tau^K$ are positively correlated with labor, while labor income taxes are positively correlated with and capital income taxes are uncorrelated with government consumption.
### US business cycle statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>1947-2008</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output $Y$</td>
<td>1.66</td>
<td>1.000</td>
<td>0.186</td>
<td>0.346</td>
<td>0.390</td>
<td></td>
</tr>
<tr>
<td>Public consumption $G$</td>
<td>3.30</td>
<td>0.186</td>
<td>1.000</td>
<td>0.083</td>
<td>0.541</td>
<td></td>
</tr>
<tr>
<td>Private consumption $C_P$</td>
<td>1.28</td>
<td>0.760</td>
<td>-0.215</td>
<td>0.256</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>Hours $L$</td>
<td>1.83</td>
<td>0.878</td>
<td>0.079</td>
<td>0.368</td>
<td>0.417</td>
<td></td>
</tr>
<tr>
<td>Capital tax $\tau_K$</td>
<td>4.75</td>
<td>0.346</td>
<td>0.083</td>
<td>1.000</td>
<td>0.497</td>
<td></td>
</tr>
<tr>
<td>Labor tax $\tau_L$</td>
<td>3.98</td>
<td>0.390</td>
<td>0.055</td>
<td>0.497</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>1956-2008</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output $Y$</td>
<td>1.53</td>
<td>1.000</td>
<td>-0.005</td>
<td>0.222</td>
<td>0.237</td>
<td></td>
</tr>
<tr>
<td>Public consumption $G$</td>
<td>1.36</td>
<td>-0.005</td>
<td>1.000</td>
<td>-0.009</td>
<td>0.152</td>
<td></td>
</tr>
<tr>
<td>Private consumption $C_P$</td>
<td>1.23</td>
<td>0.873</td>
<td>-0.169</td>
<td>0.209</td>
<td>0.127</td>
<td></td>
</tr>
<tr>
<td>Hours $L$</td>
<td>1.76</td>
<td>0.874</td>
<td>-0.169</td>
<td>0.326</td>
<td>0.366</td>
<td></td>
</tr>
<tr>
<td>Capital tax $\tau_K$</td>
<td>3.90</td>
<td>0.221</td>
<td>-0.009</td>
<td>1.000</td>
<td>0.443</td>
<td></td>
</tr>
<tr>
<td>Labor tax $\tau_L$</td>
<td>2.97</td>
<td>0.237</td>
<td>0.152</td>
<td>0.443</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>
Equilibrium in the labor market and the effect of a labor income tax

The following figure presents the effects of a labor income tax on labor supply and demand:

- The individual labor supply $l^s$ is a function of the net wage after taxes $(1 - \tau^L)w$.
- Labor demand $l^d$ is a function of the gross wage $w$.
- Taxes are imposed at a proportional tax rate $\tau^L$.
- Labor supply and demand curves are graphed as functions of the gross wage $w$.
- In the initial equilibrium at point $e$, taxes are equal to zero, $\tau^L = 0$, and the equilibrium point is $(l_0, w_0)$.
Equilibrium in the labor market and the effect of labor income tax

- If the state imposes a proportional labor income tax $\tau^L$, the labor supply curve shifts upward by a factor of $1/(1 - \tau^L)$ to $l^{s'}$.
- Tax revenues are equal to $\tau^L w_1 l_1$, which is equal to the area of the yellow rectangle $agdc$.
- While the consumer rent declines by the area $ageb$, the producer surplus falls by the area $bedc$.
- *Excess burden*: triangle $ged$. 
Equilibrium in the labor market and the effect of labor income tax

- The *economic incidence* is independent of the *legal incidence* in Walrasian labor markets.
- If labor supply (labor demand) becomes more elastic, the economic loss borne by the worker decreases (increases).
Equilibrium in the labor market and the effect of a labor income tax

\[ \text{Gross wage: } w = \frac{w_1}{(1 - \tau_L)} \]

\[ \text{Tax revenue: } acdg = abfg + bcdf \]

\[ \text{Excess burden: } gde = gfe + fde \]

Fig. 1.1 Equilibrium in the labor market and effect of a labor income tax
Welfare effects of a labor income tax

Consider the following figure:

- The optimal labor supply is found at the point where the indifference curve is tangent to the budget constraint at point $A$.
- The indifference curve is an increasing function in the $(l, Y)$-space because labor $l$ is a bad and not a good.
- An imposition of a tax $\tau^L$ results in the new equilibrium at point $C$. 
Welfare effects of a labor income tax

- The welfare loss of the household can be measured by equivalent compensation \( EV \): difference in the expenditure functions \( e(w, u) \) for the initial wage \( w^0 \) and the utility levels \( u^0 \) and \( u^1 \) associated with points \( A \) and \( B \).

\[
EV = e(w^0, u^0) - e(w^0, u^1) 
\]  

(2)

- To compute the excess burden (also called the deadweight loss) \( DWL \), we need to subtract tax revenues, \( R = \tau^L w^0 l \), from \( EV \).

- The deadweight loss is depicted by the distance between points \( C \) and \( D \).
Welfare effects of a labor income tax

\[ u_0 = u_1 = (1 - \tau_L)w_0 \]

\[ e(w_0, u_0) = I \]

\[ w^1 = (1 - \tau^1)w^0 \]

Fig. 5.10 Welfare effects of a labor income tax
Compensated (Hicksian) and Marshallian labor supply, $l$ and $L$

- The equivalent variation is equal to the area under the compensated (Hicksian) labor supply curve $h$ with utility level $u^1$.

- The compensated labor supply elasticity is given by

$$\eta_{h,w} = \frac{\Delta h / h^1}{\Delta w / w^1},$$

where $\Delta w = w^1 - w^0 = -\tau^L w^0$ denotes the change in the net wage rate.
Compensated (Hicksian) and Marshallian labor supply, $l$ and $L$

- The deadweight loss $DWL$ is equal to the surface of the triangle $BDC$:

$$DWL = EV - R = \frac{1}{2} \Delta w \cdot \Delta h = \frac{1}{2} \frac{\tau L}{1 - \tau L} \cdot \eta_{h,w} \cdot R \quad (3)$$

- Empirical value of the compensated labor supply elasticity:

$\eta_{h,w} \approx 0.30$
Compensated (Hicksian) and Marshallian labor supply, $l$ and $L$
Deadweight losses relative to revenues: Partial equilibrium

<table>
<thead>
<tr>
<th>$\tau^L$</th>
<th>$\frac{DWL}{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>1.7%</td>
</tr>
<tr>
<td>25%</td>
<td>5.0%</td>
</tr>
<tr>
<td>40% (US)</td>
<td>10.0%</td>
</tr>
<tr>
<td>50%</td>
<td>15.0%</td>
</tr>
<tr>
<td>59% (Germany)</td>
<td>21.6%</td>
</tr>
</tbody>
</table>
General Equilibrium Effects of $\tau^L$

We study the general equilibrium effects of the labor income tax rate $\tau^L$ in the Ramsey model:

- Households maximize intertemporal utility

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t),$$

(4)

$\beta$ — discount factor
General Equilibrium Effects of $\tau^L$

- Instantaneous utility:

$$u(C, 1 - L) = \frac{(C^\iota(1 - L)^{1-\iota})^{1-\sigma}}{1 - \sigma},$$  \hspace{1cm} (5)

$1/\sigma$ — intertemporal elasticity of substitution

$\iota$ and $1 - \iota$ — relative weights of consumption and leisure in utility.

- The household owns the capital stock $K_t$ in period $t$, which evolves according to

$$K_{t+1} = (1 - \delta)K_t + I_t.$$  \hspace{1cm} (6)
General Equilibrium Effects of $\tau_L$

- Capital $K_t$ depreciates at rate $\delta$.
- Household budget constraint

$$(1 - \tau_t^L)w_tL_t + r_tK_t = C_t + K_{t+1} - (1 - \delta)K_t. \quad (7)$$

The first-order conditions are represented by:

$$\lambda_t = \iota C_t^{\iota(1-\sigma)-1}(1 - L_t)^{(1-\iota)(1-\sigma)}, \quad (9a)$$

$$\lambda_t(1 - \tau_t^L)w_t = (1 - \iota)C_t^{\iota(1-\sigma)}(1 - L_t)^{(1-\iota)(1-\sigma)-1}, \quad (9b)$$

$$\lambda_t = \beta \lambda_{t+1}(1 + r_{t+1} - \delta). \quad (9c)$$

implying

$$\frac{(1 - \tau_t^L)w_t(1 - L_t)}{C_t} = \frac{1 - \iota}{\iota}$$
General Equilibrium Effects of $\tau^L$

- Goods and factor markets are characterized by perfect competition.
- Production is described by a Cobb-Douglas technology:

$$Y_t = K_t^\alpha L_t^{1-\alpha}.$$  \hspace{1cm} (10)

- Wages and the real interest rate are given by:

$$w_t = (1 - \alpha)K_t^\alpha L_t^{-\alpha},$$  \hspace{1cm} (11a)

$$r_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}.$$  \hspace{1cm} (11b)
General Equilibrium Effects of $\tau^L$

- The government budget is balanced:

$$G_t = \tau_t^L w_t L_t.$$  \hspace{1cm} (12)

- In equilibrium, the resource constraint of the economy is presented by

$$Y_t = C_t + G_t + K_{t+1} - (1 - \delta)K_t.$$  \hspace{1cm} (13)
General Equilibrium Effects of $\tau^L$

- Steady State:

\[
\frac{1}{\beta} = 1 + r - \delta, \quad (14a)
\]
\[
(1 - \tau^L)w = \frac{1 - c}{l} \frac{C}{1 - L}, \quad (14b)
\]
\[
w = (1 - \alpha)K^\alpha L^{-\alpha}, \quad (14c)
\]
\[
r = \alpha K^{\alpha - 1} L^{1 - \alpha}, \quad (14d)
\]
\[
K^\alpha L^{1-\alpha} = C + G + \delta K, \quad (14e)
\]
\[
G = \tau^L wL. \quad (14f)
\]
General Equilibrium Effects of $\tau^L$

- Dynamics in the variables $\{K_t, C_t, L_t\}$:

\[
(1 - \tau^L_t)w_t = \frac{1 - \iota}{\iota} \frac{C_t}{1 - L_t}, \tag{15a}
\]

\[
K_t^\alpha L_t^{1-\alpha} = C_t + G_t + K_{t+1} + (1 - \delta)K_t, \tag{15b}
\]

\[
\left(\frac{C_{t+1}}{C_t}\right)^{1-\iota(1-\sigma)} \left(\frac{1 - L_{t+1}}{1 - L_t}\right)^{-(1-\iota)(1-\sigma)} = \beta \left(1 + r_{t+1} - \delta\right), \tag{15c}
\]

with $r_t = \alpha(L_t/K_t)^\alpha$. 

General Equilibrium Effects of $\tau^L$

- Calibration with annual periods: $\beta = 0.96$, $\delta = 0.10$, $L = 0.30$, $\alpha = 36\%$, $\tau^L = 0.23$, $G = 0.1472 \cdot Y$, $\iota = 0.3423$, $\sigma = 2.0$.

- Computation: GAUSS program Ch5_welfare_taul.g.
General Equilibrium Effects of $\tau^L$

- Partial equilibrium effect of an increase in $\tau^L$ by 1 percentage point
  - $\tau^L$ increases from 23% to 24%.
  - Wage rate $w$ remains fixed.
  - New optimal labor supply is provided by:
    \[
    L' = 1 - \frac{1 - \rho}{\rho} \frac{C'}{(1 - \tilde{\tau}^L)w}
    \]
    \[
    C' = (1 - \tilde{\tau}^L)wL' + (r - \delta)K,
    \]
  - Old equilibrium: $(L, C) = (0.3, 0.303)$
  - New equilibrium: $(L', C') = (0.2994, 0.2998)$
General Equilibrium Effects of $\tau^L$

- *Consumption equivalent* change $\Delta$ from

$$u((1 + \Delta)C, L)) = u(C', L')$$

implying

$$\Delta = \left( \frac{u(C, 1 - L)}{u(C', 1 - L')} \right)^{-\frac{1}{\gamma(1 - \sigma)}} - 1 = -1.07\%.$$ 

- Deadweight loss = 0.000144 difference between the consumption loss, $1.07\% \times C = 0.003244$, and the additional tax revenues or, equivalently, government expenditures,

$$\Delta G = \Delta(\tau^L w L) = \tilde{\tau}^L L'w - \tau^L Lw = 0.00310$$

$$\Rightarrow \frac{DWL}{\Delta G} = \frac{0.000144}{0.00310} = 4.7\%.$$
General Equilibrium Effects of $\tau^L$

- General equilibrium effects of an increase in $\tau^L$ from 23\% to 24\%:
  - Wages and interest rates are endogenous.
  - New steady-state values: $L'' = 0.2995$ and $C''' = 0.2998$.
  - The individual’s welfare loss amounts to $\Delta = -1.10\%$
  - Tax revenue increase by $\Delta G = \Delta(\tau^L wL) = 0.00312$
  - DWL amounts to $0.0021351887$
  - $\frac{DWL}{\Delta G} = 6.8\%$ is higher in general equilibrium!
General equilibrium welfare effects of a 1% labor income tax increase

<table>
<thead>
<tr>
<th>$\tau^L$</th>
<th>$\Delta$ steady state</th>
<th>$DWL/R$</th>
<th>$\Delta$ incl. transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>-0.9%</td>
<td>3.4%</td>
<td>-0.9%</td>
</tr>
<tr>
<td>23%</td>
<td>-1.1%</td>
<td>6.8%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>40%</td>
<td>-1.3%</td>
<td>16.4%</td>
<td>-1.3%</td>
</tr>
<tr>
<td>50%</td>
<td>-1.6%</td>
<td>29.1%</td>
<td>-1.5%</td>
</tr>
<tr>
<td>59%</td>
<td>-1.9%</td>
<td>54.7%</td>
<td>-1.8%</td>
</tr>
</tbody>
</table>
General Equilibrium Effects of $\tau^L$

Next, we include the transition dynamics:

- The unexpected tax change is announced in period 0.
- Capital stock gradually declines to its new steady-state value

⇒ extra consumption during transition.
- Consumption equivalent change after accounting for the transition: $\Delta = -1.07\%$
Transition dynamics after a permanent increase in $\tau^L$
General Equilibrium Effects of $\tau^K$

**General Equilibrium Effects of a Capital Income Tax $\tau^K$**

- Household maximizes intertemporal utility

\[
U = \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)
\]

with instantaneous utility:

\[
u(C, 1 - L) = \frac{(C^\iota(1 - L)^{1-\iota})^{1-\sigma}}{1 - \sigma},\]

\[
\sigma,
\]
General Equilibrium Effects of $\tau^K$

- **Budget constraint:**

$$C_t + K_{t+1} - K_t = \begin{cases} 
(1 - \tau_t^L)w_t L_t + (1 - \tau^K)(r_t - \delta)K_t, & \text{case 1,} \\
(1 - \tau_t^L)w_t L_t + (1 - \tau^K)r_t K_t - \delta K_t, & \text{case 2.} 
\end{cases}$$

- **2 cases:**
  1. The tax law allows for the deductibility of the depreciation costs of capital.
  2. Capital depreciation is not tax-deductible.
General Equilibrium Effects of $\tau^K$

- First-order conditions:

\[
\lambda_t = \nu C_t^{(1-\sigma)-1} (1 - L_t)^{(1-\nu)(1-\sigma)},
\]

\[
\lambda_t (1 - \tau^L_t) w_t = (1 - \nu) C_t^{(1-\sigma)} (1 - L_t)^{(1-\nu)(1-\sigma)-1},
\]

\[
\lambda_t = \begin{cases} 
\lambda_{t+1} \beta \left[ 1 + (1 - \tau^K)(r_{t+1} - \delta) \right], & \text{case 1}, \\
\lambda_{t+1} \beta \left[ 1 + (1 - \tau^K)r_{t+1} - \delta \right], & \text{case 2}.
\end{cases}
\]
General Equilibrium Effects of $\tau^K$

- Goods and factor markets are characterized by perfect competition.
- Production is described by a Cobb-Douglas technology:

$$Y_t = K_t^\alpha L_t^{1-\alpha}. \quad (17)$$

- Wages and the real interest rate are given by:

$$w_t = (1 - \alpha) K_t^\alpha L_t^{-\alpha}, \quad (18a)$$
$$r_t = \alpha K_t^{-\alpha} L_t^{1-\alpha}. \quad (18b)$$

- Government budget:

$$G_t = \begin{cases} 
\tau_t^L w_t L_t + \tau^K (r_t - \delta) K_t, & \text{case 1,} \\
\tau_t^L w_t L_t + \tau^K r_t K_t, & \text{case 2,}
\end{cases} \quad (19)$$
Equilibrium conditions:

\[
\frac{1}{\beta} = \begin{cases} 
1 + (1 - \tau^k)(r - \delta), & \text{case 1}, \\
1 + (1 - \tau^k)r - \delta, & \text{case 2},
\end{cases} \tag{20a}
\]

\[
(1 - \tau^l)w = \frac{1 - \iota}{\iota} \frac{C}{1 - L}, \tag{20b}
\]

\[
w = (1 - \alpha)K^\alpha L^{-\alpha}, \tag{20c}
\]

\[
r = \alpha K^{\alpha - 1} L^{1 - \alpha}, \tag{20d}
\]

\[
K^\alpha L^{1-\alpha} = C + G + \delta K, \tag{20e}
\]

\[
G = \begin{cases} 
\tau^L wL + \tau^K(r - \delta)K, & \text{case 1}, \\
\tau^L wL + \tau^K rK, & \text{case 2}.
\end{cases} \tag{20f}
\]
General Equilibrium Effects of $\tau^K$

- Calibration: $\tau^K = 41\%$, $L = 0.3$

  $\Rightarrow \nu = 0.3355$ ($\nu = 0.3256$), $G = 20.8\%$ ($G = 29.5\%$)

- Computation: GAUSS program *Ch5_welfare_tauk.g* or MATLAB program *Ch5_welfare_tauk.m*

- Comparative steady state analysis of a change in $\tau^K$: adjustment in $\tau^L$, $G$ remains constant.
Steady-state effects of capital income tax rate $\tau^K$

case 1 (depreciation tax-deductible)
Steady-state effects of capital income tax rate $\tau^K$

Case 1 (depreciation tax-deductible)

Comparative Steady State Analysis: Case 1

- If capital taxes are abolished, $\tau^K = 0$, the labor income tax rate $\tau^L$ increases from 23.0% to 30.3%.
- For $\tau^K = 0$,
  - capital stock $K$ increases by 29%, from 0.963 to 1.245
  - labor supply $L$ falls by 4% from 0.300 to 0.289
  - output rises by 16.0%, from 0.633 to 0.734
  - consumption rises by 1.9%
  - welfare effect: 3.4% of total consumption
- Optimal $\tau^K$: it is even optimal to subsidize capital.
Steady-state effects of capital income tax rate $\tau^K$ case 2 (depreciation not tax-deductible)
Steady-state effects of capital income tax rate $\tau^K$
case 1 (depreciation not tax-deductible)

Comparative Steady State Analysis: Case 2
- The optimal capital income tax rate is zero.
- Welfare effects of an abolition of capital income taxes: 17.3% of total consumption
Chamley-Judd Result

Chamley (1986) and Judd (1985)

- **Long-run capital income taxes should be equal to zero.**

- Assumptions:
  1. Government has exogenous spending which it finances by distortionary taxes $\tau^L$ and $\tau^K$
  2. Depreciation is not tax-deductible.

- Solution to the Ramsey problem: $\tau^K = 0$ in the long-run
The model

- Households maximize utility $u(C_t, L_t)$,

$$
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[ u(C_t, L_t) + \lambda_t \left( (1 - \tau_t^L) w_t L_t 
+ (1 + (1 - \tau_t^K) r_t - \delta) K_t - C_t - K_{t+1} \right) \right]
$$
Chamley-Judd Result

First-order conditions

\[
\lambda_t (1 - \tau_t^L) w_t = -\frac{\partial u}{\partial L_t} = -u_{L_t}, \quad (21a)
\]

\[
\lambda_t = \frac{\partial u}{\partial C_t} = u_{C_t}, \quad (21b)
\]

\[
\lambda_t = \beta \lambda_{t+1} \left( 1 + (1 - \tau_{t+1}^K) r_{t+1} - \delta \right). \quad (21c)
\]

We can eliminate \( \lambda_t \) from the above equations to obtain

\[
0 = u_{L_t} + u_{C_t} (1 - \tau_t^L) w_t, \quad (22a)
\]

\[
0 = \beta u_{C_{t+1}} \left( 1 + (1 - \tau_t^K) r_{t+1} - \delta \right) - u_{C_t}. \quad (22b)
\]
Chamley-Judd Result

**Ramsey Problem**: Government maximizes the intertemporal utility of the household (4), subject to the following constraints:

1. The government budget (19) for case 2 is balanced. As a consequence, the aggregate resource constraint holds:

\[ C_t + G_t + K_{t+1} = F(K_t, L_t) + (1 - \delta)K_t \]  

(23)

2. The household maximizes intertemporal utility.

3. Firms maximize profits such that the factor prices are equal to their marginal products, \( w_t = \frac{\partial F}{\partial L_t}, \quad r_t = \frac{\partial F}{\partial K_t} \).
Chamley-Judd Result

- Government maximizes the following Lagrangian:

$$\begin{align*}
\max_{\tau_t^K, \tau_t^L, C_t, L_t, K_{t+1}} & \quad \sum_{t=0}^{\infty} \beta^t \left[ u(C_t, L_t) + \psi_t \left[ \tau_t^K F_{Kt} K_t + \tau_t^L F_{Lt} L_t - G_t \right] + \theta_t \left[ F(K_t, L_t) + (1 - \delta) K_t - C_t - G_t - K_{t+1} \right] + \mu_{1t} \left[ u_{Lt} + u_{Ct} (1 - \tau_t^L) w_t \right] + \mu_{2t} \left[ \beta u_{Ct+1} (1 + (1 - \tau_{t+1}^K) r_{t+1} - \delta) - u_{Ct} \right] \right].
\end{align*}$$ (24)

- Solution to this problem, $\{\tau_t^L, \tau_t^K\}_{t=0}^{\infty}$: Ramsey policy, $\tau^K = 0$ in the long run.
Chamley-Judd Result

- Comments on the Chamley-Judd result:
  1. The optimal wealth tax is also zero: See Problem 5.3 in my book *Solutions Manual with Problem 5.3*
  2. During the transition, the optimal capital income tax need not be zero.
     → Optimal capital income tax rate: Grüner and Heer (2000) consider a once-and-for-all change in $\tau^K$ and include an analysis of the transition dynamics
     ⇒ optimal tax rate $\tau^K = 9\%$. 
Indeed, it is optimal to tax capital in the first period as much as possible

- Intertemporal government budget constraint:

\[
\sum_{t=0}^{\infty} \left[ \frac{\tau_t^L w_t L_t + \tau_t^K r_t K_t}{\prod_{s=0}^{t} (1 + r_s)} \right] = \sum_{t=0}^{\infty} \left[ \frac{G_t}{\prod_{s=0}^{t} (1 + r_s)} \right], \tag{25}
\]

where we set \( r_0 \equiv 0 \) and \( K_0 \) is given.

⇒ Time-inconsistency of zero capital income taxes
Laffer curve

\[ R \]

\[ \tau \]

\[ \tau_0 \quad \tau_1 \quad \bar{\tau} \quad \tau_2 \quad \tau_3 \quad \tau_{100} \quad \tau \]
Laffer Curve

The model of Trabandt and Uhlig (2011):

- Households maximize intertemporal utility

\[ U_0 = \sum_{t=0}^{\infty} \beta^t [u(C_t, L_t) + \varphi(G_t)], \quad \text{(26)} \]

where instantaneous utility is a function of consumption \( C \) and labor \( L \):

\[ u(c, l) = \frac{1}{1 - \sigma} \left( C^{1-\sigma} \left[ 1 - \nu_0 (1 - \sigma) L^{1+1/\nu_1} \right]^\sigma - 1 \right). \quad \text{(27)} \]
Laffer Curve

- Capital accumulation:

\[ K_{t+1} = (1 - \delta)K_t + I_t. \] (28)

- The household holds two forms of assets, government bonds \( B_t \) and capital \( K_t \) with returns \( r^b_t \) and \( r_t \).

- Budget constraint:

\[
(1+\tau_C^t)C_t + I_t + B_{t+1} = (1-\tau^L_t)w_t L_t + (1-\tau^K_t)(r_t - \delta)K_t + \delta K_t + (1+r^B_t)B_t + Tr_t, \quad (29)
\]

where \( \tau^C_t \) and \( Tr_t \) denote the constant consumption tax rate and government transfers to the households in period \( t \).

- Depreciation is tax-deductible.
Laffer Curve

The first-order conditions are represented by

\[
\lambda_t (1 + \tau_C^t) = C_t^{-\sigma} \left[ 1 - \nu_0 (1 - \sigma) L_t^{1 + 1/\nu_1} \right]^{\sigma},
\]

\[
\lambda_t (1 - \tau_L^t) w_t = \nu_0 \sigma \left( 1 + \frac{1}{\nu_1} \right) C_t^{1 - \sigma} \left[ 1 - \nu_0 (1 - \sigma) L_t^{1 + 1/\nu_1} \right]^{\sigma - 1} L_t^{1/\nu_1},
\]

\[
\lambda_t = \beta \lambda_{t+1} \left[ 1 + (1 - \tau^K_{t+1}) (r_{t+1} - \delta) \right],
\]

\[
\lambda_t = \beta \lambda_{t+1} (1 + r_{t+1}^B).
\]

From (30c) and (30d), it follows that the two assets \( B_t \) and \( K_t \) must yield the same return after taxes,

\[
r_{t+1}^B = (1 - \tau^K_{t+1}) (r_{t+1} - \delta).
\]
Laffer Curve

- Production is Cobb-Douglas in the two production factors, capital $K_t$ and labor $L_t$:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}. \quad (31)$$

- Total factor productivity $A_t$ grows at the exogenous rate $\gamma_A$:

$$A_t = A_0 (1 + \gamma_A)^t. \quad (32)$$

- Stationary growth factor: $\psi = (1 + \gamma_A)^{1/(1-\alpha)}$
Laffer Curve

⇒ stationary output is given by:

\[ \tilde{Y}_t \equiv \frac{Y_t}{\psi^t} = \frac{A_0 (1 + \gamma_A)^t K_t^\alpha L_t^{1-\alpha}}{\psi^t} = A_0 \tilde{K}_t^\alpha L_t^{1-\alpha}, \] (33)

where \( \tilde{K}_t \equiv K_t/\psi^t \).

• Factor market equilibrium with competitive goods and factor markets:

\[ \tilde{w}_t = (1 - \alpha) \frac{\tilde{Y}_t}{L_t}, \] (34a)

\[ r_t = \alpha \frac{\tilde{Y}_t}{\tilde{K}_t}. \] (34b)
Laffer Curve

- Government
  - The government finances government expenditures with taxes and debt:

\[ G_t + Tr_t + r_t^B B_t = T_t + B_{t+1} - B_t, \]  
  \hspace{1cm} (35)

- Taxes \( T_t \) are given by:

\[ T_t = \tau^C C_t + \tau^L w_t L_t + \tau^K (r_t - \delta) K_t. \]  
  \hspace{1cm} (36)
Laffer Curve

- Equilibrium conditions:
  - Goods market:
    \[ Y_t = C_t + G_t + I_t. \]  \( (37) \)
  - The real return on bonds in the long run is determined by the Euler condition:
    \[ 1 + r^B = \frac{\psi^\sigma}{\beta}. \]

⇒ As a consequence, the real interest rate \( r \) and, hence, the capital-labor ratio \( \tilde{K}/L \) and wage \( w \) depend on the capital income tax rate \( \tau^K \) but not on the labor income tax rate \( \tau^L \) according to:

\[ 1 + (1 - \tau^K)(r - \delta) = \frac{\psi^\sigma}{\beta}. \]
Laffer Curve

- Calibration as in Trabandt and Uhlig (2011): $r^B = 0.04$, $\nu_1 = 1.0$, $1/\sigma = 1/2.0$, $\tau^L = 0.28$, $\tau^K = 0.36$, $\tau^C = 0.05$, $\psi = 1.02$, $\alpha = 0.38$, $\delta = 0.07$, $B/Y = 63\%$, $G/Y = 18\%$, $L = 0.25 \Rightarrow \nu_0 = 3.732$.

- Computation: GAUSS program Ch5_laffer.g
US Laffer curve: Labor income tax rate $\tau^L$
US Laffer curve: Labor income tax rate $\tau^L$

Laffer curve for the labor income tax rate $\tau^L$ (holding the other taxes $\tau^C$ and $\tau^K$ constant):

- At the benchmark with $\tau^L = 28\%$, labor income tax revenues (the solid red line) are equal to 0.08278, which amounts to 17.4\% of GDP.
- Labor income tax revenues peak at $\tau^L = 71\%$ and can be increased by 67\%.
- The effect of $\tau^L$ on total revenues $T_t$ is illustrated by the broken green line.

$\rightarrow$ Total tax revenue increases by a maximum of 54.5\% = 9.5\% of present GDP
US Laffer curve: Capital income tax rate $\tau^K$
US Laffer curve: Capital income tax rate $\tau^K$

Laffer curve for the capital income tax rate $\tau^K$ (holding the other taxes $\tau^C$ and $\tau^L$ constant):

- Capital income tax revenue peaks at $\tau^K = 91\%$.
- Capital income tax only exhibits limited potential for increasing tax revenue.
Growth effects of taxes


  ➔ Regressions are subject to estimation problems (e.g., endogeneity problem).

- Arnold (2008) finds that the distortions and the forgone economic growth from taxation increase in the following order:

  1. Property taxes
  2. Consumption tax
  3. Personal income tax
  4. Corporate income tax
Growth effects of taxes

Endogenous Growth with Government Expenditures: Barro (1990)

- Government expenditures enter the production function and increase productivity.
- Production $F(K_t, L_t, G_t)$ in period $t$ uses capital $K_t$, labor $L_t$, and public services $G_t$ as inputs according to:

\[ Y_t = F(K_t, L_t, G_t) = AL_t^{1-\alpha} K_t^\alpha G_t^{1-\alpha}. \] (39)

→ constant returns to scale in the private inputs $K_t$ and $L_t$

→ Euler’s theorem applies.
Growth effects of taxes

→ constant returns to scale in the inputs $K_t$ and $G_t$ (unbounded factors)
→ endogenous growth

• The representative firm maximizes profits $\Pi_t$ in period $t$

$$\Pi_t = [1 - \tau] AL_t^{1-\alpha} K_t^\alpha G_t^{1-\alpha} - r_t K_t - w_t L_t,$$

where $w$ and $r$ denote the real wage and interest rate, respectively.

• Production is taxed at rate $\tau$. 
Growth effects of taxes

- First-order conditions of the firm:

\[ w_t = (1 - \tau)F_L = (1 - \tau)(1 - \alpha)AL_t^{-\alpha}K_t^\alpha G_t^{1-\alpha}, \quad (41a) \]

\[ r_t = (1 - \tau)F_K = (1 - \tau)\alpha AL_t^{1-\alpha}K_t^{\alpha-1} G_t^{1-\alpha}. \quad (41b) \]

- In equilibrium, profits are zero.

- Government budget is balanced in each period \( t \):

\[ G_t = \tau Y_t. \quad (42) \]
Growth effects of taxes

- Household maximizes its intertemporal utility

\[
U = \sum_{t=0}^{\infty} \beta^t u(C_t),
\]  

(43)

where \( \beta < 1 \) denotes its discount factor.

- Instantaneous utility:

\[
u(C_t) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma},
\]

(44)

where \( 1/\sigma \) denotes the intertemporal elasticity of substitution.

- Household labor supply is exogenous and given by \( L_t = L \).
Growth effects of taxes

- Household budget:

\[ C_t + K_{t+1} = w_t L + (1 + r_t - \delta) K_t. \]  \hspace{1cm} (45)

- Euler equation:

\[ \left( \frac{C_{t+1}}{C_t} \right)^\sigma = \beta (1 + r_{t+1} - \delta). \]  \hspace{1cm} (47)
Growth effects of taxes

- Growth rate of consumption $\gamma^C$:

$$\frac{C_{t+1} - C_t}{C_t} = \{ \beta [1 + r_{t+1} - \delta] \}^{\frac{1}{\sigma}} - 1$$

$$= \beta^{\frac{1}{\sigma}} \left[ 1 + (1 - \tau)\alpha A^{\frac{1}{\alpha}} \left[ L \tau^{\frac{1-\alpha}{\alpha}} - \delta \right] \right]^{\frac{1}{\sigma}} - 1.$$  \hspace{1cm} (49)

⇒ maximum consumption growth rate is attained for a production tax $\tau = 1 - \alpha$:

$$\gamma^C = \beta^{\frac{1}{\sigma}} \left[ 1 + \alpha^2 (1 - \alpha) A^{\frac{1-\alpha}{\alpha}} L^{\frac{1-\alpha}{\alpha}} - \delta \right]^{\frac{1}{\sigma}} - 1.$$  \hspace{1cm} (50)
Growth effects of taxes

⇒ For this value of $\tau$, the marginal product of the public input is equal to one and, therefore, equal to its marginal costs:

$$\frac{\partial F(K_t, G_t, L)}{\partial G_t} = (1-\alpha) \frac{Y_t}{G_t} = (1-\alpha) \frac{Y_t}{\tau Y_t} = \frac{1-\alpha}{1-\alpha} = 1.$$

- In a competitive equilibrium, all variables $G$, $K$, and $C$ grow at the same rate $\gamma$. 
Growth effects of taxes

- **Pareto Efficiency**

Benevolent dictator maximizes (43) subject to the resource constraint

\[ C_t + G_t + K_{t+1} = AL_t^{1-\alpha} K_t^\alpha G_t^{1-\alpha} + (1 - \delta) K_t. \quad (53) \]

The first-order conditions are given by

\[ \lambda_t = C_t^{-\sigma}, \quad (54a) \]

\[ \lambda_t = \beta \lambda_{t+1} \left( 1 + \alpha AL_t^{1-\alpha} K_t^{\alpha-1} G_t^{1-\alpha} - \delta \right), \quad (54b) \]

\[ 1 = (1 - \alpha) AL_t^{1-\alpha} K_t^\alpha G_t^{-\alpha}. \quad (54c) \]
Growth effects of taxes

implying the maximum growth rate $\tilde{\gamma}^C$:

$$\frac{C_{t+1} - C_t}{C_t} \equiv \tilde{\gamma}^C = \left[ \beta \left( 1 + \alpha (1 - \alpha)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} L^{\frac{1-\alpha}{\alpha}} - \delta \right) \right]^{\frac{1}{\sigma}} - 1.$$ 

$\Rightarrow$ Pareto-efficient growth rate $\tilde{\gamma}^C$ is larger than the growth-maximizing rate $\gamma^C$ in eq. (50) for the case of a decentralized economy!
Growth effects of taxes


- Human capital accumulation drives economic growth: The more time people spend on education, the higher the economic growth rate will be.
- Opportunity costs of learning are inversely related with labor income tax levels.

⇒ higher capital income taxes are associated with higher growth!
Growth effects of taxes

- Production
  - Firms use both physical and human capital $K_t$ and $H_t$ as production inputs:

$$Y_t = A_0 \left( \alpha K_t^{\rho_p} + (1 - \alpha)(u_t H_t)^{\rho_p} \right)^{1/\rho_p}, \quad (55)$$

$$\sigma_p = 1/(1 - \rho_p) \quad \text{— CES in production}$$

$$u_t \quad \text{— working time of the household}$$
Growth effects of taxes

- In factor market equilibrium, the factor prices are equal to their marginal products:

\[
\begin{align*}
  w_t &= A_0 (1 - \alpha) \left( \alpha K_t^{\rho_p} + (1 - \alpha)(u_t H_t)^{\rho_p} \right)^{\frac{1}{\rho_p} - 1} (u_t H_t)^{\rho_p - 1}, \\
  r_t &= A_0 \alpha \left( \alpha K_t^{\rho_p} + (1 - \alpha)(u_t H_t)^{\rho_p} \right)^{\frac{1}{\rho_p} - 1} (K_t)^{\rho_p - 1}.
\end{align*}
\] (56a, 56b)

\( w \) — wage per efficiency unit \( uH \).
Growth effects of taxes

- Human capital accumulation:
  - Individuals spend $v_t$ units of time on education, and human capital $h_t$ accumulates according to:

  $$h_{t+1} = h_t + Dv_t \bar{h}_t,$$

  (57)

  $\bar{h}_t$ — average human capital in the economy.

- The household takes $\bar{h}_t$ as given.
Growth effects of taxes

- Households
  - The number of households is equal to one.
  - The representative household maximizes intertemporal utility:

\[
\sum_{t=0}^{\infty} \beta^t \left[ C_t (1 - u - v)^{\iota} \right]^{1-\sigma} \]

subject to the budget constraint in period \( t \)

\[
(1 - \tau^L_t) w_t u_t h_t + (1 + (1 - \tau^K_t) r_t - \delta) K_t + tr_t = C_t + K_{t+1}. \]

1 — leisure
\( \tau^L \) — labor income tax
\( \tau^K \) — capital income tax
\( tr \) — government transfers
Growth effects of taxes

- First-order conditions with respect to $C_t$, $K_{t+1}$, $u_t$, $v_t$, and $h_{t+1}$ are presented by:

\[
\lambda_t = C_t^{1-\sigma} (1 - u_t - v_t)^{\nu(1-\sigma)},
\]

(60a)

\[
\lambda_t = \beta \lambda_{t+1} (1 + (1 - \tau_{t+1}^K) r_{t+1} - \delta),
\]

(60b)

\[
\lambda_t (1 - \tau_t^L) w_t h_t = \nu C_t^{1-\sigma} (1 - u_t - v_t)^{\nu(1-\sigma)-1},
\]

(60c)

\[
\mu_t = \beta \left[ \mu_{t+1} + \lambda_{t+1} (1 - \tau_{t+1}^L) w_{t+1} u_{t+1} \right].
\]

(60e)
Growth effects of taxes

- Government: balanced budget

\[ G_t + Tr_t = \tau_t^L w_t u_t H_t + \tau_t^K r_t K_t. \]  

(61)

- Equilibrium

- Resource constraint of the economy:

\[ Y_t = K_{t+1} - (1 - \delta)K_t + C_t + G_t. \]  

(62)

- Both aggregate human capital \( H_t \) and average human capital \( \bar{h}_t \) are equal to individual human capital \( h_t \):

\[ H_t = \bar{h}_t = h_t. \]  

(63)

- Aggregate transfers are equal to individual transfers:

\[ Tr_t = tr_t. \]  

(64)
Growth effects of taxes

- **Steady State:**
  - The aggregate variables $Y_t$, $H_t$, $K_t$, and $C_t$ all grow at the endogenous growth rate $\gamma$.
  - Time allocation of the household is constant, $u_t = u$ and $v_t = v$.
  - The steady state is described by the following 9 equations in the 9 endogenous variables $\tilde{K} = K/H$, $\tilde{Y} = Y/H$, $u$, $v$, $\tilde{C} = C/Y$, $\gamma$, $r$, $w$, and $\tau^L$:

\[
\frac{(1 + \gamma)^\sigma}{\beta} = 1 + (1 - \tau^K)r - \delta, \quad (68a)
\]
\[
\gamma = Du^s, \quad (68b)
\]
\[
\frac{(1 + \gamma)^{\sigma - 1}}{\beta} = 1 + w D u^{s-1}, \quad (68c)
\]
Growth effects of taxes

\[(1 - \tau^L)w = \iota \frac{\tilde{C}}{1 - u - v},\]  
\[r = A_0 \alpha \left( \alpha \tilde{K}^{\rho_p} + (1 - \alpha)u^{\rho_p} \right)^{\frac{1}{\rho_p} - 1} \tilde{K}^{\rho_p - 1},\]  
\[w = A_0 (1 - \alpha) \left( \alpha \tilde{K}^{\rho_p} + (1 - \alpha)u^{\rho_p} \right)^{\frac{1}{\rho_p} - 1} u^{\rho_p - 1},\]  
\[\tilde{Y} = (\gamma + \delta) \tilde{K} + \tilde{C} + \tilde{G},\]  
\[\tilde{Y} = A_0 \left( \alpha \tilde{K}^{\rho_p} + (1 - \alpha)u^{\rho_p} \right)^{\frac{1}{\rho_p}},\]  
\[\tilde{G} + \tilde{T}r = \tau^K r K + \tau^L w u.\]
Growth effects of taxes

- Calibration follows Lucas (1990): $\rho_p = -2/3 \implies \sigma_p = 0.6$,
  $\alpha = 0.361$, $\gamma = 1.5\%$, $\zeta = 0.8$, $\delta = 8.0\%$, $A_0 = 1$, $\beta = 0.96$,
  $1/\sigma = 1/2$, $u = 30\%$, $G/Y = 19\%$
  $\tau^K = 41\%$ and $\tau^L = 28\%$ as above.

  $\bar{v} = 0.0628$, $D = 0.137$, $\iota = 5.327$, $Tr/Y = 13.1\%$,
  $\tilde{Tr} = 0.0435$, $\tilde{G} = 0.0631$, and $\tilde{Y} = 0.322$

- Computation: Gauss program Ch5_lucas.g.
Growth rate effects of capital income taxation $\tau^K$
Steady-state effects of capital income taxation $\tau^K$
Growth rate effects of capital income taxation $\tau^K$

- For a reduction in the capital income tax rate $\tau^K$ from 41% to 0%, the economic growth rate falls by 4.6%, from 1.50% to 1.43%.

- Optimal capital income tax rate: Grüner and Heer (2000)
  - consider a once-and-for-all change in $\tau^K$
  - include an analysis of the transition dynamics
  $\Rightarrow$ optimal tax rate $\tau^K = 9\%$. 
RBC Model with Stochastic Taxes

RBC model with stochastic capital and labor income tax rates:

- helps to improve the modeling of empirical business cycle effects, e.g., the low correlation of wages and employment
- McGrattan (1994): stochastic taxes help to explain the volatility of output, investment, and hours of work.
- Gomme, Ravikumar, and Rupert (2011): can explain nearly 80% of the volatility of the return to capital.
RBC Model with Stochastic Taxes

The Model

- **Households**
  - maximize expected value of intertemporal utility

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t), \quad 0 < \beta < 1, \tag{69}
\]

- instantaneous utility \( u(., .) \):

\[
u(C_t, 1 - L_t) = \left(\frac{C_t^u (1 - L_t)^{1-\iota}}{1 - \sigma}\right)^{1-\sigma} \tag{70}\]
RBC Model with Stochastic Taxes

- Effective consumption is presented by the CES aggregator of private consumption $C^p_t$ and government consumption $G_t$:

$$C = \left[ \phi \left( C^p \right)^{1-1/\rho_c} + (1-\phi) G^{1-1/\rho_c} \right]^{1-1/\rho_c}.$$

- The household owns the capital stock, which evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t. \quad (71)$$

- Capital depreciates at rate $\delta$.
- The budget constraint of the individual is presented by

$$C^p_t + I_t = (1-\tau^L_t)w_tL_t + (1-\tau^K_t)r_tK_t + \tau^K_t \delta K_t + tr_t. \quad (72)$$
RBC Model with Stochastic Taxes

- First-order conditions:

\[
\lambda_t = \nu \phi C_t^{\nu(1-\sigma)-1} (1 - L_t)^{(1-\nu)(1-\sigma)} (\Xi_t)^{1-1/\rho_c} - \frac{1}{\rho_c} (C_t^p)^{-1/\rho_c},
\]

(73a)

\[
\lambda_t (1 - \tau_t^L) w_t = (1 - \nu) C_t^{\nu(1-\sigma)} (1 - L_t)^{(1-\nu)(1-\sigma)-1},
\]

(73b)

\[
\lambda_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \left[ 1 + (1 - \tau_{t+1}^K) (r_{t+1} - \delta) \right] \right\},
\]

(73c)

with \(\Xi_t\) defined as

\[
\Xi_t \equiv \phi (C_t^p)^{1-1/\rho_c} + (1 - \phi) G_t^{1-1/\rho_c}.
\]
RBC Model with Stochastic Taxes

- **Production:**
  - Production $Y_t$ is characterized by constant returns to scale in labor $L_t$ and capital $K_t$:
    \[
    Y_t = Z_t K_t^\alpha L_t^{1-\alpha}.
    \]  

- Production is subject to a technology shock $Z_t$ that is governed by the following AR(1) process:
  \[
  \ln Z_t = \rho Z \ln Z_{t-1} + \epsilon_t^Z, \quad \epsilon_t^Z \sim N(0, \sigma^Z),
  \]  

- In a factor market equilibrium, factors are compensated by their marginal products:
  \[
  w_t = (1 - \alpha) Z_t K_t^\alpha L_t^{-\alpha},
  \]  
  \[
  r_t = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}.
  \]
RBC Model with Stochastic Taxes

- **Government:**
  - $G_t$ follows a first-order autoregressive process:
    \[
    \ln G_t = (1 - \rho^G) \ln G + \rho^G G_{t-1} + \epsilon^G_t, \quad \epsilon^G_t \sim N(0, \sigma^G), \tag{77}
    \]
    where $G$ denotes steady-state government consumption.

- The labor and capital income taxes also follow stochastic processes as in Gomme, Ravikumar, and Rupert (2011):
  \[
  \ln \tau^K_t = (1 - \rho^K) \ln \tau^K + \rho^K \tau^K_{t-1} + \epsilon^K_t, \quad \tag{78}
  \]
  \[
  \ln \tau^L_t = (1 - \rho^L_1 - \rho^L_2) \ln \tau^L + \rho^L_1 \tau^L_{t-1} + \rho^L_2 \tau^L_{t-2} + \epsilon^L_t. \quad \tag{79}
  \]
  with $\epsilon^K_t \sim N(0, \sigma^K)$ and $\epsilon^L_t \sim N(0, \sigma^L)$. 

RBC Model with Stochastic Taxes

- Government budget is balanced in each period $t$:

$$Tr_t = \tau_t^L w_t L_t + \tau_t^K (r_t - \delta)K_t - G_t.$$  \hspace{1cm} (80)

- General equilibrium: Resource constraint of the economy

$$Y_t = C_t^p + G_t + I_t.$$  \hspace{1cm} (81)
RBC Model with Stochastic Taxes

- **Calibration**
  - Calibration of the preference parameters as in Chapter 4: \( \phi = \frac{3}{4} \) and \( \rho_c = 0.5 \)
  - Calibration of the stochastic tax parameter as in Gomme, Ravikumar, and Rupert (2011): \( \tau^K = 32\% \), \( \tau^L = 28\% \), \( \rho^K = 0.9725 \), \( \rho^{L1} = 0.7841 \), and \( \rho^{L2} = 0.2047 \).
  - Autoregressions of the HP-filtered components of the (logarithmic) two tax rates \( \tau^K \) and \( \tau^L \), of order 1 and 2, respectively, results in standard deviations of the residual equal to \( \sigma^K = 0.027 \) and \( \sigma^L = 0.028 \).

- **Computation**: GAUSS program *Ch5_RBC_stoch_tax.g*
Impulse responses to a capital income tax shock

- Output
- Capital Tax
- Capital Tax Shock

- Effective Consumption
- Investment
- Government Consumption

- Hours
- Real Wage
- Real Interest Rate

- Private Consumption
- Capital Stock
RBC Model with Stochastic Taxes

Impulse responses to a capital income tax shock

- Shock on $\tau^K$ of 1 std dev: capital income tax rate $\tau^K_t$ increases from 32.00% to 32.86%.

$\Rightarrow$ Higher capital income taxes reduce investment, and thus, income is shifted from savings into consumption.

- The total effect on demand is negative, and output declines.

- The household also reduces its labor supply.

- Since the effect of reduced labor supply dominates the effect of a smaller capital stock, the marginal product of capital (labor) decreases (increases), and therefore, the interest rate (wage) falls (rises).
Impulse responses to a labor income tax shock

- **Output**: Shows a significant initial decrease followed by a recovery.
- **Labor Tax**: Remains largely unchanged except for a slight decrease.
- **Labor Tax Shock**: Sharp decrease followed by a quick return to baseline.

- **Effective Consumption**: Slightly decreases but recovers over time.
- **Investment**: Shows a dip initially followed by a gradual increase.
- **Government Consumption**: Remains stable with minor fluctuations.

- **Hours**: Initially decreases sharply and then stabilizes.
- **Real Wage**: Slightly decreases initially and then stabilizes.
- **Real Interest Rate**: Decreases sharply initially and then stabilizes.
- **Capital Tax**: Remains relatively stable with minor fluctuations.

- **Private Consumption**: Initially decreases sharply and then stabilizes.
- **Capital Stock**: Decreases initially and then stabilizes.
Impulse responses to a labor income tax shock

- Shock on $\tau^L$ of 1 std dev: $\tau^L_t$ increases from 28.00% to 28.78%.
  - households work less, and hours decline by 0.7%.
  - both output and income fall, and the household has to reduce both consumption and savings.
  - investment falls.
- the marginal product of labor (equal to the wage $w$) rises, while the real interest rate $r$ declines.
Second moments for the RBC model with stochastic taxes

<table>
<thead>
<tr>
<th>Variable</th>
<th>$s_x$</th>
<th>$s_x/s_Y$</th>
<th>$r_{xY}$</th>
<th>$r_{xL}$</th>
<th>$r_{xG}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output $Y$</td>
<td>1.30</td>
<td>1.00</td>
<td>1.00</td>
<td>0.78</td>
<td>0.22</td>
</tr>
<tr>
<td>Private consumption $C^p$</td>
<td>0.58</td>
<td>0.45</td>
<td>0.84</td>
<td>0.85</td>
<td>0.27</td>
</tr>
<tr>
<td>Investment $I$</td>
<td>4.50</td>
<td>3.46</td>
<td>0.95</td>
<td>0.61</td>
<td>-0.05</td>
</tr>
<tr>
<td>Hours $L$</td>
<td>1.00</td>
<td>0.77</td>
<td>0.78</td>
<td>1.00</td>
<td>0.45</td>
</tr>
<tr>
<td>Real wage $w$</td>
<td>0.81</td>
<td>0.62</td>
<td>0.63</td>
<td>0.02</td>
<td>-0.20</td>
</tr>
<tr>
<td>Real interest rate $r$</td>
<td>1.35</td>
<td>1.04</td>
<td>0.96</td>
<td>0.79</td>
<td>0.21</td>
</tr>
<tr>
<td>Public consumption $G$</td>
<td>1.23</td>
<td>0.94</td>
<td>0.22</td>
<td>0.45</td>
<td>1.00</td>
</tr>
<tr>
<td>Capital tax $\tau^K$</td>
<td>3.38</td>
<td>2.60</td>
<td>-0.06</td>
<td>-0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>Labor tax $\tau^L$</td>
<td>3.06</td>
<td>2.35</td>
<td>-0.35</td>
<td>-0.63</td>
<td>0.00</td>
</tr>
</tbody>
</table>
RBC Model with Stochastic Taxes

Time Series Results

- Output, investment, and consumption are more volatile.
- Consumption and investment are also more volatile relative to output, which is in better accordance with empirical observations.
- Both tax rates $\tau^K$ and $\tau^L$ are negatively correlated with labor supply $L$, exhibiting correlation coefficients equal to -0.15 and -0.63, respectively.

⇒ the positive correlation of wages with employment falls to 0.02 (from 0.55 in the case without stochastic taxes) and is in much better accordance with empirical evidence.


