New Keynesian Q-Targeting with Epstein Zin Utility

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Abstract

I study the relevance of explicit asset price targeting within a medium scale New Keynesian economy that features preference shocks and a general recursive utility representation as in Epstein and Zin (1989) next to a Carlstrom and Fuerst (1997)-type financial friction. While the monetary authority is found to optimally respond negatively to asset price movements, the welfare enhancement of doing so with respect to a purely anti-inflationary regime is virtually negligible. Besides, the empirical performance of the model is found to critically hinge on the imposed zero lower interest bound.
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1 Introduction

The recent financial crisis has reactivated interest in the question of whether central banks should react to asset price movements. One strand of this literature is concerned with potential benefits of augmenting a Taylor (1993)-type interest rate rule by an explicit asset price indicator. E.g., recently, Faia and Monacelli (2007) directly compare different Taylor rule scenarios by means of their respective welfare implications within a New Keynesian (NK) framework that features a Carlstrom and Fuerst (1998)-type financial friction. They find that central banks should react inversely to asset price movements, however with hardly any effect on welfare. Heer and Maußner (2014) confirm this result in a related framework. Thereby, they use Calvo (1983)-stickiness, for both nominal prices and wages, and derive their neoclassic (Tobin (1969)’s) q-target directly from capital adjustment costs.

Both mentioned papers argue that their analysis would benefit from a further generalization of their models’ shock structure. This motive is further strengthened by the empirical evidence reported in Gali, Smets, and Wouters (2011) who emphasize on the importance of demand side shocks in the recent Great Recession. In the present paper, I follow this suggestion in that I add two preference shocks to the Heer and Maußner (2014) framework, one to the atemporal marginal rate of substitution (MRS) between consumption and leisure and another one to the impatience parameter, the latter being found of particular relevance in the Gali, Smets, and Wouters analysis. Moreover, I generalize the utility representation to the Kreps-Porteous class of Epstein and Zin (1989). With these changes, I both re-examine the above authors’ results with respect to optimal monetary policy and also analyze the effect on empirical performance. Besides, employing the Epstein and Zin representation allows me to discuss my findings on the relevance of the risk aversion parameter and the elasticity of intertemporal substitution in front of the recent literature.²

While preference shocks are constantly used in order to improve the performance of macroeconomic models in explaining real world empirical ev-

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¹See the introductory section of Heer and Maußner (2014) for a short review on the literature.
²See Tallarini (2000), Rudebusch and Swanson (2012), and Gourio (2012). Note that the Epstein and Zin lifetime utility representation introduces additional flexibility as it loosens the strict entanglement of risk aversion and intertemporal substitution behavior that characterizes the standard representation of additively separable expected utility.
doubts on the theoretical soundness of this approach have been raised, questioning the suitability of models featuring these kinds of “dubiously structural” elements. However, the two considered preference shifts are supported by recent theoretical work. On the one hand, following the work of [Hall (1997), Nakajima (2005)] provides a microfoundation for the MRS shift showing that a representative agent model that features such a shock “can be viewed as a reduced form of a heterogeneous-agents economy with incomplete markets.” On the other hand, [Gourio (2012)] provides a very appealing microfoundation for the discounting shock as he shows that time varying impatience parameters may be interpreted as a reduced form of a model that features a time varying probability of some economic disaster which plays a critical role in the agents’ (relative) assessment of investment alternatives.

The present paper’s approach is numerical. I use a second order perturbation to approximate the consequences for welfare that are associated with the examined monetary policy regimes. I thereby only consider interest rate rules that are operational in the sense of [Schmitt-Grohe and Uribe (2004a)]. In particular, such rules may only include observable macroeconomic variables as arguments, must not exceed a strict upper threshold on the probability of the equilibrium interest rate violating the zero lower bound, and must induce a unique rational expectations equilibrium. The welfare assessment follows the [Lucas (1987)] tradition in computing relative gains with respect to some benchmark scenario as employed by [Schmitt-Grohe and Uribe (2004a)] and subsequently adopted by [Faia and Monacelli (2007)] and Heer and Maußner (2014) in their q-targeting analyses. The fact that [Tallarini (2000)] documents considerably higher welfare costs of business cycle fluctuations in Epstein and Zin environments made up for the second motivation to accordingly generalize the lifetime utility representation.

With respect to empirical performance, I find that allowing the preference shocks to fully exert their respective potential leads to a dramatic improvement in the model’s data fit with respect to the [Heer and Maußner (2014)] model. This, however, is accompanied by a large and implausible proba-

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3E.g. [Hall (1997)] argues in an important contribution that empirically observed labor market fluctuations may be largely attributed to shifts in the MRS.
4See [Chari, Kehoe, and McGrattan (2009)], e.g. p. 244.
5See [Nakajima (2005)], p. 1334.
6Cf. [Schmitt-Grohe and Uribe (2004)].
7The distance between the empirical evidence and the simulation results can be reduced
bility of a zero lower interest bound violation. In particular, once a rea-
sonably restrictive upper boundary on this probability is imposed, the im-
provement in empirical performance is negligible. On the role of the risk
aversion parameter, I find that it plays virtually no role in the absence of
the discounting shock but has considerable impact on the equity premium in
its presence while exercising a very discreet effect on business cycle figures.
While the first finding can be seen as an extension to the preliminary discus-
sion in Heiberger and Ruf (2014a), the second is akin to results reported in
Tallarini (2000) and Rudebusch and Swanson (2012).

With respect to the potential benefits of q-targeting, the results confirm
the main findings of Faia and Monacelli (2007) and Heer and Maußner (2014).
While the central bank is advised to react negatively to asset price move-
ments, the welfare effects of q-targeting are virtually negligible. The result is
neither particularly sensitive to the calibration nor to the allowed zero lower
bound probability. Hence, together with the above mentioned research, it
provides a robust basis for the conjecture that there cannot be made a strong
point for q-targeting solely through further extension of the shock structure
or the list of frictions.

The remainder of the paper is organized as follows. Section 2 gives a
detailed description of the model while Section 3 is concerned with the ac-
tual simulation results, i.e. the framework’s empirical performance and its
assessment of q-targeting regimes, and thus summarizes the essential con-
tribution of the paper. Section 4 concludes. Throughout, I delegate more
tedious derivations to the appendix.

2 Description of the model

The model economy features three sectors, a consumption sector, a produc-
tive sector, and the government. I start with a non-formal overview before I
lay out the particular assumptions explicitly. Note that time is discrete and
the planning horizon is infinite, i.e. \( t \in \mathbb{N} \).

**Consumption sector** Households purchase consumption goods, save via
bonds and capital, and supply differentiated labor services and capital to
the productive sector. Due to monopolistic competition in the labor market,
households execute market power which is in turn limited by nominal wage stickiness. They derive utility from leisure and consumption (beyond its habit level) and are assumed to be representable by one stand-in agent who maximizes his recursive lifetime utility.

**Productive sector** The economy’s output is produced through labor and capital inputs. The differentiated good for consumption, investment and government spending is produced subject to a financial constraint. More precisely, the productive sector borrows from the household to finance its factor expenses. Thereby, agency costs arise from asymmetric information in the credit relationship. Monopolistic competition in the good’s market gives rise to price setting power which is again constraint by nominal stickiness. Moreover, capital accumulation is subject to real adjustment costs which generate a time varying real price of capital, Tobin’s $q$.

**Public sector** The government conducts both fiscal and monetary policy. First, it exogenously purchases public consumption financed through taxes and government bonds. Second, it endogenously controls those bonds’ nominal interest rate through a [Taylor](#) type feedback rule that features an explicit asset price target ($q$). Via the household’s Euler-Lucas equation for these bond’s real interest, this rule impacts the real economy due to the presence of the above outlined distortions.

### 2.1 Consumption sector

In any period $t$, the representative household supplies the productive sector with a continuum of labor services $N_t(k), k \in [0, 1]$, which are not perfectly substitutable. There is thus room for wage setting, which is limited, however, by [Calvo (1983)](#) type stickiness.

To ease the equilibrium analysis in the aggregate labor market, I first introduce a labor agency that bundles any period’s supplied labor services to match the aggregate labor demand $N^d_t$ of the productive sector.
2.1.1 Labor agency

The labor agency bundles according to

\[ N_t^d = \left( \int_0^1 N_t(k) \frac{\epsilon_{n-1}}{\epsilon_n} dk \right)^\frac{\epsilon_n}{\epsilon_{n-1}}, \epsilon_n > 1. \]

The agency sells this bundle at the nominal wage index \( W_t \) while it pays the respective labor types their nominal wages \( W_t(k) \). The optimal labor demand for supply of type \( k \) services \( N_t^s(k) \), given wages, thus satisfies

\[ N_t^s(k) = \arg \max_{N_t(k)} \left\{ W_t N_t^d - \int_0^1 W_t(k) N_t(k) dk \right\} = \left( \frac{W_t(k)}{W_t} \right)^{-\epsilon_n} N_t^d. \]

The labor agency is an abstract modeling instance which I thus require to not receive any economic rents. I.e.

\[ W_t N_t^d \overset{!}{=} \int_0^1 W_t(k) N_t^s(k) dk. \]

This determines the nominal wage index

\[ W_t = \left( \int_0^1 W_t(k)^{1-\epsilon_n} dk \right)^\frac{1}{1-\epsilon_n}. \]

2.1.2 Representative household

Within any period \( t \), the representative household derives utility from a Cobb-Douglas composite good

\[ u_t := (C_t - C_t^h)^\nu (1 - N_t^s)^{\epsilon_n (1-\nu)}, \nu \in (0, 1), \]

that aggregates consumption \( C_t \) and leisure, \( 1 - N_t^s \), where \( N_t^s \) is the household’s period \( t \) aggregate labor supply (normalized to a maximum of 1)

\[ N_t^s := \int_0^1 N_t^s(k) dk = N_t^d \int_0^1 \left( \frac{W_t(k)}{W_t} \right)^{-\epsilon_n} dk. \]

\(^8\)Note that this deviates from the analysis in Heer and Maußner (2014). I choose this composite good to be able to carry out a similar welfare assessment of the considered monetary policy regimes in presence of the more general lifetime utility representation described below.
Furthermore, the assumed habit formation in consumption is exogenous in the sense of Campbell and Cochrane (1999) and follows
\[ C_h^t := \chi C_{t-1}, \chi \in (0, 1), \]
and the marginal rate of substitution of consumption for leisure
\[ \text{MRS}_t := e^{\theta_t} \frac{1 - \nu C_t - C_h^t}{1 - N_t^s}, \]
is subject to an exogenous stationary AR(1) shift
\[ \theta_{t+1} = \rho \theta_t + \sigma_{\theta_t} \epsilon_{t+1}^\text{MRS}, \rho \in [0, 1), \sigma_{\theta} \geq 0 \]  \hfill (1)
and \( \epsilon_{t}^\text{MRS} \sim \text{iidN}(0, 1) \).

Next, I assume the household to have a recursive utility representation over temporal composite good lotteries of the Kreps-Porteus class introduced by Epstein and Zin (1989)
\[ U_t = \left[ (1 - \beta)u_t^{1 - \psi} + \beta_t \left( E_t \left[ U_{t+1}^{1 - \gamma} \right] \right)^{\frac{1 - \psi}{1 - \gamma}} \right]^{1 - \frac{1}{\psi}}, \beta \in (0, 1), 1 \neq \psi > 0, 1 \neq \gamma > 0, \]
where impatience
\[ \beta_t := \beta e^{\vartheta_t} \]
is time variable due to another exogenous stationary AR(1) shift
\[ \vartheta_{t+1} = \rho_{\vartheta} \vartheta_t + \sigma_{\vartheta_t} \epsilon_{t+1}^\text{BETA}, \rho_{\vartheta} \in [0, 1), \sigma_{\vartheta} \geq 0 \]  \hfill (2)
and \( \epsilon_{t}^\text{BETA} \sim \text{iidN}(0, 1) \).

The household’s period t budget constraint reads
\[ \int_0^1 \frac{W_t(k)}{P_t} N_s(k) dk + r_t K_t^H + D_t + (Q_t - 1) \frac{B_t}{P_t} - T_t \geq C_t + q_t (K_{t+1}^H - (1 - \delta) K_t^H) + \frac{B_{t+1} - B_t}{P_t}, \]
where \( P \) denotes the price of the consumption good, \( K^H \) the household’s capital savings (i.e. the part of the real capital stock that is owned by the household), \( r \) is its respective real rental rate and \( q \) the real price of capital. Furthermore \( D \) is real dividends, \( B \) is the household’s nominal bond holdings and \( Q \) the nominal gross bond return. \( T \) denotes real taxes.

As in the flexible price taking benchmark, the household chooses his capital stock and bond holdings optimally, given their returns. Also note that in a utility maximum, consumption must exhaust the budget constraint such that one can plug in for \( C_t \) in the objective. I begin with the necessary optimality conditions with respect to the investment alternatives \( K_{t+1}^H \) and \( B_{t+1} \).
**Savings**  
First,

\[ q_t = \mathbb{E}_t [m_{t+1,t} (r_{t+1} + q_{t+1}(1 - \delta))] \tag{3}, \]

is the representative household’s Euler-Lucas equation for the equity asset, where

\[ m_{t+1,t} = \beta_t \left( \frac{U_{t+1}}{(\mathbb{E}_t [U_{t+1}])^{1-\gamma}} \right)^{1-\gamma} \left( \frac{u_{t+1}}{u_t} \right)^{1-\gamma} \frac{C_t - C^h_t}{C_{t+1} - C^h_{t+1}} \]

is his stochastic discount factor from \( t + 1 \) to \( t \). Accordingly, \( m_{t+\tau,t} \) denotes the respective stochastic discount factor for general \( \tau \geq 0 \), with \( m_{t,t} \equiv 1 \).

Second, with respect to government bonds, optimal saving requires

\[ 1 = \mathbb{E}_t \left[ \frac{m_{t+1,t} Q_{t+1}}{\pi_{t+1}} \right] = \mathbb{E}_t \left[ \frac{m_{t+1,t}}{\pi_{t+1}} \right] Q_{t+1}, \tag{4} \]

with \( \pi_{t+1} := \frac{P_{t+1}}{P_t} \) denoting inflation from \( t \) to \( t + 1 \).

**Wage setting**  
I assume labor services to be provided under monopolistic competition. Additionally, I assume nominal wages to be sticky. In particular, I employ the Calvo (1983) wage setting mechanism. i.e. in each period the representative household is only allowed to optimally set the nominal wage for an exogenous fraction of \( (1 - \varphi_n) \times 100\% \) of his labor services while the remaining \( \varphi_n \times 100\% \) are simply adjusted to previous period inflation. Formally, for arbitrary \( k \in [0, 1] \),

\[ W_t(k) = \begin{cases} 
\hat{W}_t(k), & \text{with probability } 1 - \varphi_n \\
W_{t-1}(k)\pi_{t-1}, & \text{with probability } \varphi_n,
\end{cases} \]

where \( \hat{W}_t(k) \) denotes the optimally set nominal wage for labor type \( k \) in period \( t \) and, of course, \( \varphi_n \in [0, 1] \). More generally, in \( \tau \in \mathbb{N} \) periods ahead, the nominal wage depends on \( \hat{W}_t(k) \) only if no further reoptimizing is possible until then, thus

\[ W_{t+\tau}(k) = \hat{W}_t(k) \prod_{j=0}^{\tau-1} \pi_{t+j}, \text{ with probability } (1 - \varphi_n)\varphi_n^\tau. \]

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\(^9\)See the derivation of the optimal wage equation in the appendix for how this stochastic discount factor emerges in the present economy.
Consequently, with probability $(1 - \varphi_n)\varphi_n^\tau$

\[
\frac{W_{t+\tau}(k)}{W_{t+\tau}} = \frac{\tilde{W}_t(k) \prod_{j=0}^{\tau-1} \pi_{t+j}}{w_{t+\tau} P_t \prod_{j=1}^{\tau} \pi_{t+j}} = \frac{\tilde{w}_t(k) \pi_t}{w_{t+\tau} \pi_{t+\tau}},
\]

where \(\tilde{w}\) denotes the optimally chosen real wages. As in Schmitt-Grohe and Uribe (2005), I assume the representative household to consider the equilibrium processes for \(N^d, \pi\) and also the wage index \(W\) as given.

Now, in \(t\), the representative household chooses the nominal wage that maximizes his lifetime utility \(U_t\) for every labor type \(k \in [0, 1]\) which is drawn by the Calvo lottery. Since the problem turns out to be identical for all such \(k\), I write

\[
\tilde{W}_t := \tilde{W}_t(k), \forall t,
\]

and accordingly

\[
\tilde{w}_t := \frac{\tilde{W}_t}{P_t}, \forall t.
\]

From its necessary optimality condition,

\[
\frac{\partial U_t}{\partial \tilde{w}_t} = 0,
\]

one can then derive\textsuperscript{10} the following expression for period \(t\)’s optimal wage

\[
\tilde{w}_t = \frac{\epsilon_n}{\epsilon_n - 1} \frac{\Delta_{1,t}}{\Delta_{2,t}},
\]

where \(\Delta_{1,t}\) and \(\Delta_{2,t}\) satisfy

\[
\Delta_{1,t} = N_t^d w_t^\epsilon_n \text{MRS}_t + \mathbb{E}_t \left[ m_{t+1,t} \varphi_n \left( \frac{\pi_t}{\pi_{t+1}} \right)^{-\epsilon_n} \Delta_{1,t+1} \right]
\]

\[
\Delta_{2,t} = N_t^d w_t^\epsilon_n + \mathbb{E}_t \left[ m_{t+1,t} \varphi_n \left( \frac{\pi_t}{\pi_{t+1}} \right)^{1-\epsilon_n} \Delta_{2,t+1} \right].
\]

The nonprofit condition of the labor agency further allows the derivation of an equation that links the processes of the optimal wage and the aggregate

\textsuperscript{10}The derivation is carried out in the appendix. There it is also shown how the chosen lifetime utility specification generalizes the analysis in Heer and Maußner (2014).
wage index. It makes use of a law of large numbers argument and introduces $A_t \subset [0,1]$ as the set of labor types for which wages are allowed to be set optimally in period $t$.

$$W_t^{1-\epsilon_n} = \int_0^1 W_t(k)^{1-\epsilon_n} dk =$$

$$= \int_{A_t} W_t^{1-\epsilon_n}(k) dk + \int_{[0,1] \setminus A_t} W_t^{1-\epsilon_n}(k) dk =$$

$$= (1 - \varphi_n) \tilde{W}_t^{1-\epsilon_n} + \pi_{t-1}^{1-\epsilon_n} \int_{[0,1] \setminus A_t} W_{t-1}^{1-\epsilon_n}(k) dk =$$

$$= (1 - \varphi_n) \tilde{W}_t^{1-\epsilon_n} + \varphi_n (\pi_{t-1} W_{t-1})^{1-\epsilon_n},$$

where the last equality follows from the assumption that it holds for the Calvo lottery that the probability of being allowed to optimally adjust prices in some period $t$ is independent of period $(t - 1)$’s wage. Thus, by a law of large numbers, the average last period wage over all who are not chosen to optimize today equals the overall last period’s average wage. In real terms, this reads

$$w_t^{1-\epsilon_n} = (1 - \varphi_n) \tilde{w}_t^{1-\epsilon_n} + \varphi_n \left( \frac{\pi_{t-1}}{\pi_t} w_{t-1} \right)^{1-\epsilon_n}. \quad (7)$$

Eventually, I introduce a variable for the factor of proportionality between the aggregate labor supply and demand, i.e.

$$s_t^n := \int_0^1 \left( \frac{W_t(k)}{\tilde{W}_t} \right)^{-\epsilon_n} dk,$$

which can be interpreted as a measure of wage dispersion, such that

$$N_t^s = s_t^n N_t^d.$$ Again using the wage index condition, one can derive (cf. appendix)

$$s_t^n = (1 - \varphi_n) \left( \frac{\tilde{w}_t}{w_t} \right)^{-\epsilon_n} + \varphi_n \left( \frac{w_{t-1} \pi_{t-1}}{w_t \pi_t} \right)^{-\epsilon_n} s_{t-1}^n. \quad (8)$$

### 2.2 Productive sector

The productive sector consists of two subsectors plus two abstract agencies (akin to the labor agency above) which are there to ease exposition. While
one agency produces the capital goods, another one bundles the differentiated production goods to meet the aggregate demand for (private and public) consumption and capital investment.

### 2.2.1 Primary production

In the first step, there is a continuum of firms with unit mass indexed by $f \in [0,1]$ producing the not yet differentiated economy output. Such a firm’s period $t$ real net worth is

$$NW_t(f) := (q_t(1 - \delta) + r_t) X_t(f) + \Delta_t(f),$$

where $X_t(f)$ denotes firm $f$’s real capital stock and $\Delta_t(f)$ is a real transfer from the household to firm $f$. Such a firm produces from labor (working hours) $N^P_t(f)$ and real capital services $K_t(f)$. It is further assumed to have a Cobb-Douglas technology with an idiosyncratic element to its stochastic total factor productivity. Formally,

$$Y^P_t(f) := \omega_t(f) e^{Z_t(1 - \alpha) (K_t(f))^{\alpha}}, \alpha \in (0, 1),$$

where for all $f \in [0,1]$ and all $t$ it holds for the idiosyncratic shock that

$$\omega_t(f) \sim \text{iid}, \text{ with density } \phi, \text{ distribution } \Phi \text{ and } \mathbb{E}[\omega_t(f)] = \Omega_t,$$

and

$$Z_{t+1} = \rho_Z Z_t + \sigma_Z \varepsilon^\text{TFP}_{t+1}, \rho_Z \in [0, 1), \sigma_Z \geq 0$$

and

$$\varepsilon^\text{TFP}_t \sim \text{iidN}(0, 1).$$

Now, in $t$, any firm $f$ has to solve both a static factor allocation and an intertemporal asset accumulation problem. I start with the former which the firm solves in a two stage procedure.

First, after the realization of $Z_t$ but still before the realization of $\omega_t(f)$ it already has to commit itself to its real factor expenses $M_t(f)$. Second, given its first step decision, it decides over the respective optimal factor inputs once $\omega_t(f)$ is realized. I start with the description of this second step.

**Optimal factor allocation** With $q_t$ as the price of the primary good relative to the retail price of the final good, any firm $f \in [0,1]$ chooses its
factor inputs to maximize real revenue subject to the first stage commitment to $M_t(f)$. I.e. formally

$$\max_{N^P_t(f), K^P_t(f)} g_t \omega_t(f) e_t^Z \left( N^P_t(f) \right)^{1-\alpha} (K_t(f))^\alpha$$

s.t. $M_t(f) \geq w_t(f)N^P_t(f) + r^P_t(f)K_t(f)$,

where $r^P_t$ is the real rental rate on capital services to be paid by any firm in the primary sector. Note that for a pair $(N^P_t(f), K^P_t(f))$ to be optimal it must necessarily fully exhaust the side restriction. Thus, with $\lambda_t(f)$ as firm $f$’s Langrange multiplier to the period $t$ constraint, this problem’s first order conditions read

$$g_t \omega_t(f) e_t^Z (1 - \alpha) \left( N^P_t(f) \right)^{-\alpha} (K_t(f))^\alpha = \lambda_t(f)w_t$$  \hspace{1cm} (10)$$

and

$$g_t \omega_t(f) e_t^Z \alpha \left( N^P_t(f) \right)^{1-\alpha} (K_t(f))^{\alpha - 1} = \lambda_t(f)r^P_t.$$  \hspace{1cm} (11)$$

Thus, with

$$k_t(f) := \frac{K_t(f)}{N^P_t(f)}$$

it follows that

$$\frac{w_t}{r_t^P} = \frac{1}{\alpha} k_t(f)$$

and hence

$$k_t(f) = k := \frac{\alpha w_t}{1 - \alpha r_t^P}, \forall f \in [0, 1].$$

Further, with

$$v_t(f) := \frac{\lambda_t(f)}{\omega_t(f)}$$

it also follows from any of the two necessary conditions that

$$v_t(f) = v_t, \forall f \in [0, 1].$$

The interpretation of $v_t$ is a markup on real factor costs which again follows from the above first order conditions. Precisely, multiply (10) by $N^P_t(f)$, multiply (11) by $K_t(f)$, and then add both equations to find

$$g_t e_t^Z N^P_t(f) k_t^\alpha = v_t M_t(f).$$
With

\[ N_t := \int_0^1 N_t^P(f)df \]

and

\[ M_t := \int_0^1 M_t(f)df \]

this directly translates to the aggregate sector level

\[ g_t e^{Z_t} k_t^\alpha = v_t M_t. \]  \hspace{1cm} (12)

Next, note that since both \( k_t \) and \( M_t(f) \) are independent of the realization of \( \omega_t(f) \), it follows from

\[ M_t(f) = (w_t + r^P_t k_t) N_t^P(f) \]

that also

\[ N_t^P(f) = \frac{M_t(f)}{w_t + r^P_t k_t} \]

is independent of the realization of \( \omega_t(f) \). Therefore, also

\[ K_t(f) \equiv k_t N_t^P(f) \]

and

\[ e^{Z_t} k_t^\alpha N_t^P(f) \]

are independent of the realization of \( \omega_t(f) \)\(^{11}\). Thus, the aggregate primary production

\[ Y_t^P := \int_0^1 Y_t^P(f)df \equiv \int_0^1 \omega_t(f)e^{Z_t} k_t^\alpha N_t^P(f)df \]

can be written as

\[ Y_t^P = e^{Z_t} k_t^\alpha \int_0^1 \omega_t(f)N_t^P(f)df \]

such that it follows by the above established independence of \( N_t(f) \) and \( \omega_t(f) \) that

\[ Y_t^P = \int_0^1 \omega_t(f)df \left( e^{Z_t} k_t^\alpha \int_0^1 N_t^P(f)df \right) \equiv \int_0^1 \omega_t(f)df \left( e^{Z_t} k_t^\alpha N_t \right). \]

\(^{11}\)Note the independence of \( \epsilon_t^{\text{TPP}} \) and \( \omega_t(f) \).
With
\[ \tilde{Y}_t := e^{Z_t} k_t^\alpha N_t \]
and by a law of large numbers argument saying \( \int_0^1 \omega_t(f) df = \Omega_t \), it thus follows that
\[ Y_t^P := \Omega_t \tilde{Y}_t. \] (13)

Eventually, define
\[ K_t := k_t N_t \]
and restate the first order conditions solely in terms of aggregate variables. I.e.
\[ w_t = \frac{g_t}{v_t} (1 - \alpha) \frac{\tilde{Y}_t}{N_t} \] (14)
and
\[ r_t^P = \frac{g_t}{v_t} \alpha \frac{\tilde{Y}_t}{K_t}. \] (15)

I next describe decision making on the first stage, i.e. committing to real factor expenses \( M_t(f) \).

**Optimal factor expenses** Any firm \( f \in [0, 1] \) has the possibility to borrow from the representative household in case its internal budget is not sufficient to finance its chosen factor expenses, i.e. in real terms
\[ NW_t(f) < M_t(f). \]

The associated loan is intraperiod to a real gross interest of \( r_t^L \). Further, the real credit volume is
\[ \max \{ 0; M_t(f) - NW_t(f) \} \]
and the payback is agreed as
\[ \min \left\{ r_t^L (M_t(f) - NW_t(f)); \omega_t(f) e^{Z_t} k_t^\alpha N_t^P (f) \right\}, \]
where the second payback scenario displays the reported bankruptcy of firm \( f \) in which the credit contract demands \( f \) to pay back as much as possible. Given \( M_t(f) \geq NW_t(f) \) and \( r_t^L \), one can define a bankruptcy threshold of idiosyncratic productivity
\[ \bar{\omega}_t(f) := \frac{r_t^L (M_t(f) - NW_t(f))}{g_t e_t^p k_t^p N_t^P (f)}. \]
Accordingly, period \( t \)'s credit contract is fully specified as soon as two of the three variables \( r^L_t, M_t(f) \) and \( \bar{\omega}_t(f) \) are determined.

Next, I follow Carlstrom and Fuerst (1997) and assume the borrower-lender relationship to be characterized by the following asymmetric information set up. First, the idiosyncratic productivity is private knowledge of the firm such that it has an incentive to untruthfully misreport his real revenue. Second, this moral hazard problem is overcome by monitoring. Specifically, whenever a firm \( f \) declares bankruptcy, the representative household verifies its productivity. Such monitoring is assumed to be exact but costly, with real costs proportional to the marked up real factor costs \( \nu_t M_t(f) \) by factor \( \kappa \in (0, 1) \).\(^{12}\)

Moreover, note that since the representative household lends to all firms in the primary sector, his lending portfolio is fully diversified. He thus acts as if he was risk neutral. His expected return from the credit relationship with firm \( f \) is

\[
\int_0^{\bar{\omega}_t(f)} g_t e^{Z_t} N^P_t(f) k^\alpha_t \phi(\omega) d\omega + \int_{\bar{\omega}_t(f)}^\infty r^L_t (M_t(f) - NW_t(f)) \phi(\omega) d\omega - \int_0^{\bar{\omega}_t(f)} \kappa v_t M_t(f) \phi(\omega) d\omega
\]

\[
= g_t e^{Z_t} N^P_t(f) k^\alpha_t \left( \int_0^{\bar{\omega}_t(f)} \omega \phi(\omega) d\omega + (1 - \Phi(\bar{\omega}_t(f))) \bar{\omega}_t(f) - \kappa \Phi(\bar{\omega}_t(f)) \right)
\]

\[
= g_t e^{Z_t} N^P_t(f) k^\alpha_t g(\bar{\omega}_t(f)),
\]

with

\[g(x) := \int_x^\infty \omega \phi(\omega) d\omega + (1 - \Phi(x)) x - \kappa \Phi(x),\]

On the other hand, firm \( f \)'s expected return from taking the contract opportunity, i.e. for \( M_t(f) > NW_t(f) \), accordingly reads

\[
\int_{\bar{\omega}_t(f)}^\infty \left( \omega g_t e^{Z_t} N^P_t(f) k^\alpha_t - r^L_t (M_t(f) - NW_t(f)) \right) \phi(\omega) d\omega
\]

\[
= g_t e^{Z_t} N^P_t(f) k^\alpha_t f(\bar{\omega}_t(f)),
\]

\(^{12}\)See Carlstrom and Fuerst (1998) for remarks on the optimality of such monitoring.

\(^{13}\)Note that for the original case (cf. Carlstrom and Fuerst (1997)) of \( \Omega_t = 1, \forall t, g(\cdot) \) has the interpretation of the lender’s share of aggregate real revenue. Further note that the second equality follows from the stochastic independence of \( e^{Z_t P} \) and \( \omega \).
with
\[ f(x) := \int_x^{\infty} \omega \phi(\omega) d\omega - (1 - \Phi(x)) x^{14} \]

Now, as in Carlstrom and Fuerst (1997), by assumption, the borrower receives all economic rents concerning the credit relationship. Thus, given \( NW_t(f) \), in order for the contract \((\bar{\omega}_t(f), M_t(f))\) to be optimal it has to hold that
\[
(\bar{\omega}_t(f), M_t(f)) = \arg \max_{x,y} \ g_t e^{z_t} F_t^P(f) k_t^\alpha f(x) \\
\text{s.t.} \ g_t e^{z_t} F_t^P(f) k_t^\alpha g(x) = y - NW_t(f),
\]
or equivalently \(^{15}\)
\[
(\bar{\omega}_t(f), M_t(f)) = \arg \max_{x,y} \ v_t y f(x) \\
\text{s.t.} \ y = \frac{NW_t(f)}{1 - v_t g(x)}; \tag{16}
\]
with first order conditions
\[
M_t(f) = \frac{NW_t(f)}{1 - v_t g(\bar{\omega}_t(f))}
\]
and
\[
1 = v_t \left( \Omega_t - \kappa \Phi(\bar{\omega}_t(f)) - \frac{f(\bar{\omega}_t(f)) \kappa \Phi(\bar{\omega}_t(f))}{1 - \Phi(\bar{\omega}_t(f))} \right) \tag{16}
\]
Note that the latter equation determines the optimal bankruptcy threshold \(\bar{\omega}_t(f)\) in dependence of \(v, \kappa\) and the distribution of \(\omega_t(f)\), which are all independent of \(f\). It hence follows for all \(t\) that
\[
\bar{\omega}_t(f) = \bar{\omega}_t, \forall f \in [0, 1].
\]
Thus, with
\[
NW_t := \int_0^1 NW_t(f) df
\]
\(^{14}\)Note again that for the case of \(\Omega_t = 1, \forall t\), \(f(\cdot)\) has the interpretation of the borrower’s share of aggregate real revenue. \(^{15}\)This uses \(g_t e^{z_t} F_t^P(f) k_t^\alpha = v_t M_t(f)\) as derived above. \(^{16}\)This condition is derived in the appendix.
one can restate the necessary conditions for an optimal contract in terms of aggregate variables only, i.e.

\[ M_t = \frac{NW_t}{1 - v_t g(\bar{\omega}_t)} \]  

(17)

and

\[ 1 = v_t \left( \Omega_t - \kappa \Phi(\bar{\omega}_t) - \frac{f(\bar{\omega}_t) \kappa \phi(\bar{\omega}_t)}{1 - \Phi(\bar{\omega}_t)} \right). \]  

(18)

Eventually, note that

\[ f(\bar{\omega}_t) + g(\bar{\omega}_t) = \Omega_t - \kappa \Phi(\bar{\omega}_t) \]

such that, in the aggregate, the moral hazard situation leads to a destruction of real sectoral revenue, i.e.

\[ g_t e^{\bar{z}_t} k_t^a N_t (f(\bar{\omega}_t) + g(\bar{\omega}_t)) = g_t \int_0^1 Y_t^P(f) df - \kappa \Phi(\bar{\omega}_t) g_t \bar{Y}_t. \]

This waste thereby depends positively on the monitoring costs and the probability of bankruptcy.

In the next paragraph, I describe how the firm finally chooses its next period capital stock.

**Optimal asset accumulation** Any firm \( f \in [0, 1] \) is owned by the representative household to which consequently all net profits are distributed in form of dividend payments \( D_t^P(f) \). Thus, the value of the firm to its owner is directly derived from the evaluation of future dividend payments. Thereby, next to the common TFP process \( \{Z_t\} \), in every period \( t \) there is a common threshold \( \bar{\omega}_t \), due to the credit contract described above, such that whenever the random idiosyncratic productivity \( \omega_t(f) \) falls below this level, firm \( f \) goes bankrupt. According to the latter, in every period there are two dividend payment scenarios, namely

\[
D_t^P(f) := \begin{cases} 
  g_t \omega_t(f) e^{\bar{z}_t} k_t^a N_t^P(f) - r_t^L (M_t(f) - NW_t(f)) - \Delta_t(f) - q_t X_{t+1}(f), & \text{if } \omega_t(f) > \bar{\omega}_t \\
  -\Delta_t(f) - q_t X_{t+1}(f), & \text{if } \omega_t(f) \leq \bar{\omega}_t.
\end{cases}
\]
Therefore, in some period $t$, the expected dividend payment with respect to $\omega_t(f)$ of some firm $f \in [0, 1]$ given $Z_t$ is

$$
\mathbb{E}_{\omega_t(f)} \left[ D^P_t(f) \right] = \int_{\tilde{\omega}_t}^{\infty} (g_t(\omega_t(f)) e^{Z_t k_t^\alpha} N^P_t(f) - r_t^L (M_t(f) - NW_t(f))) \phi(\omega) d\omega + \\
- \Delta_t(f) - q_t X_{t+1}(f) \\
= v_t M_t(f) f(\tilde{\omega}_t) - \Delta_t(f) - q_t X_{t+1}(f).
$$

Furthermore, the representative household is assumed to be less patient in his role as some firm $f$’s shareholder such that future dividend payments are discounted via

$$
m^{\eta}_{t+\tau,t} := \eta^\tau m^{\eta}_{t+\tau,t}, \eta \in (0, 1).$$

The value of firm $f$ to its owner, i.e. the expected discounted sum of future dividends from firm $f$, accordingly reads

$$
fv_t(f) := \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m^{\eta}_{t+\tau,t} D^P_{t+\tau}(f) \right]
$$

$$
= \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m^{\eta}_{t+\tau,t} \mathbb{E}_{\omega_{t+\tau}} \left[ D^P_{t+\tau}(f) \right] \right]
$$

$$
= \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m^{\eta}_{t+\tau,t} (v_t M_t(f) f(\tilde{\omega}_t) - \Delta_t(f) - q_t X_{t+1}(f)) \right],
$$

where the second equality follows from the independence of $\omega_{t+i}(f)$ and $\epsilon^\text{TFP}_{t+j}$ for all $i, j \in \mathbb{N}$. Now, in some period $t$, firm $f$’s management chooses next period’s capital stock $X_{t+1}$ in order to maximize this firm value. After substituting for $M_{t+\tau}(f)$ from the first order condition of the contract and also plugging in the definition of $NW_t$, it follows that $f$’s capital investment Euler equation reads

$$
q_t = \mathbb{E}_t \left[ m^{\eta}_{t+\tau,t} \frac{v_{t+1} f(\tilde{\omega}_{t+1})}{1 - v_{t+1} g(\tilde{\omega}_{t+1})} (q_{t+1}(1 - \delta) + r_{t+1}) \right].
$$

\textbf{17} This is assumed in order to ensure the relevance of the financial constraint. See Carlstrom and Fuerst (1997), p. 897 et seq., for explanatory remarks. Following Chugh (2013), an alternative way to interpret such stronger discounting is to think of it as capturing asymmetric information between the firms management and its owners (households).
To conclude, define
\[ D_P^t := \int_0^1 D_P^t(f)df \]
for the aggregate dividend payment in the primary goods sector and
\[ \Delta_t := \int_0^1 \Delta_t(f)df \]
for the aggregate real transfer from the household to primary production.

### 2.2.2 Secondary production

The so far produced primary goods are understood as inputs to the next step towards the final good. In this step, an intermediary firm produces a continuum of differentiated outputs \( Y_t(j), j \in [0, 1] \) from the primary input goods to satisfy the demand side preferences. It is assumed that these differentiated goods are only qualitative modifications of the primary goods and that there is limited substitutability between them giving rise to monopolistic competition. Again, the corresponding price setting is assumed to be restricted to the Calvo (1983) mechanism.

Further, I again employ an abstract agency to describe the aggregation of those intermediary goods to one final good that meets the economy’s demand \( Y_t^d \).

**Final good agency** The final good agency bundles according to
\[ Y_t^d = \left( \int_0^1 Y_t(j)^{\frac{1}{1-\epsilon_y}} dj \right)^{\frac{1}{\epsilon_y-1}}, \epsilon_y > 1, \]
and sells this bundle at the nominal price index \( P_t \) while it pays the nominal price \( P_t(j) \) for the respective intermediary goods. The optimal demand for supply of intermediary good \( j, Y_t^*(j) \), given prices thus satisfies
\[ Y_t^*(j) = \arg \max_{Y_t(j)} \left\{ P_t Y_t^d - \int_0^1 P_t(j) Y_t(j) dj \right\} = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y} Y_t^d. \]

I again require the abstract agency to not receive any economic rents,
\[ P_t Y_t^d = \frac{1}{\epsilon_y} \int_0^1 P_t(j) Y_t^*(j) dj, \]
such that the nominal price index for the final good is determined by

\[ P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_y} dj \right)^{\frac{1}{1-\epsilon_y}}. \]

**Intermediary production** In some period \( t \), the representative firm in the intermediary production sector buys primary goods to produce and has a real profit with respect to some product \( j \in [0, 1] \) of

\[ \frac{P_t(j)}{P_t} Y^*_t(j) = \left( \left( \frac{P_t(j)}{P_t} \right)^{1-\epsilon_y} - g_t \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y} \right) Y^*_t, \]

As above, I assume the Calvo (1983) mechanism to describe the price setting of the intermediary goods. Thus, for some arbitrary \( j \in [0, 1] \) it holds that

\[ P_t(j) = \begin{cases} P_{t-1}(j) \pi_t, & \text{with probability } \varphi_y, \\ \tilde{P}_t(j), & \text{with probability } 1 - \varphi_y, \end{cases} \]

where \( \tilde{P}_t(j) \) denotes the optimally set nominal price for intermediary good \( j \) and, of course, \( \varphi_y \in [0, 1] \). More generally, in \( \tau \in \mathbb{N} \) periods ahead, the nominal price depends on \( P_t(j) \) only if no further reoptimizing is possible until then, thus

\[ P_{t+\tau}(j) = \tilde{P}_t(j) \prod_{j=0}^{\tau-1} \pi_{t+j}, \text{ with probability } (1 - \varphi_y) \varphi_y^{\tau}. \]

Consequently, with probability \( (1 - \varphi_y) \varphi_y^{\tau} \)

\[ \frac{P_{t+\tau}(j)}{P_{t+\tau}} = \frac{\tilde{P}_t(j) \prod_{j=0}^{\tau-1} \pi_{t+j}}{P_t \prod_{j=1}^{\tau} \pi_{t+j}} = \tilde{p}_t(j) \frac{\pi_t}{\pi_{t+\tau}}, \]

where, for all \( j \) and \( t \), \( \tilde{p}_t(j) := \frac{\tilde{P}_t(j)}{P_t} \). As in the wage setting framework described above, I assume the representative intermediary goods producer to consider the equilibrium processes for \( Y^d, \pi \) and the price index \( P \) as given.
Now, for all intermediary goods $j$ that are drawn by the Calvo lottery, the management of the representative firm in the intermediary goods sector chooses $\tilde{p}_t(j)$ such that it maximizes its firm value

$$E_t \left[ \sum_{\tau=0}^{\infty} m_{t+r,t} D^I_{t+r} \right],$$

with

$$D^I_t := \int_0^1 D^I_t(j)\,dj.$$

This problem’s first order condition is again identical for all $j$ such that

$$\tilde{p}_t(j) = \tilde{p}_t,$$

with

$$\tilde{p}_t = \frac{\epsilon_y}{\epsilon_y - 1} \Gamma_{1,t},$$

where $\Gamma_{1,t}$ and $\Gamma_{2,t}$ satisfy

$$\Gamma_{1,t} = g_t Y^d_t \pi_{t}^{\epsilon_y} + \mathbb{E}_t [m_{t+1,t} \varphi_y \Gamma_{1,t+1}],$$

$$\Gamma_{2,t} = Y^d_t \pi_{t}^{\epsilon_y - 1} + \mathbb{E}_t [m_{t+1,t} \varphi_y \Gamma_{2,t+1}].$$

Furthermore, analogously to the description of the wage setting above, one can derive the following equation that links the nominal price index to the optimal nominal price

$$P_t^{1-\epsilon_y} = (1 - \varphi_y) P_t^{1-\epsilon_y} + \varphi_y (\pi_{t-1} P_{t-1})^{1-\epsilon_y}$$

and thus, in real terms,

$$1 = (1 - \varphi_y) P_t^{1-\epsilon_y} + \varphi_y \left( \frac{\pi_{t-1}}{\pi_t} \right)^{1-\epsilon_y}.$$ \hspace{1cm} (22)

Eventually, I introduce a variable for the factor of proportionality between the aggregate supply and demand for intermediate production. I.e., with

$$Y^s_t := \int_0^1 Y^s_t(j)\,dj,$$

The derivation of this condition is found in the appendix.
I write

\[ Y_t^s = s_t^y Y_t^d, \]

where

\[ s_t^y := \int_0^1 \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_y} dj \]

can be interpreted as a measure of price dispersion. Accordingly, again from the price index equation it finally follows that (cf. appendix)

\[ s_t^y = (1 - \varphi_y) \tilde{p}_{t-1}^{-\epsilon_y} + \varphi_y \left( \frac{\pi_t}{\pi_t-1} \right)^{-\epsilon_y} s_{t-1}^y. \tag{23} \]

### 2.2.3 Capital goods agency

To keep the exposition clearly arranged, I introduce yet another non-profit unit in the production sector, the capital goods agency. It separately describes the production of the economy’s capital stock. This agency produces period \( t + 1 \) capital goods from capital investment \( I_t \) and the current stock \( K_t \). This production is subject to adjustment costs. More precisely, the production of next period’s capital stock is described as

\[ \Psi \left( \frac{I_t}{K_t} \right) K_t, \]

where \( \Psi \) is a strictly increasing and concave function that satisfies

\[ \Psi(\delta) = \delta \]

and

\[ \Psi'(\delta) = 1 \]

such that there are no adjustment costs when investment solely replaces material wear

\[ \delta K_t, \delta \in [0, 1]. \]

Now, in some period \( t \) the capital goods agency maximizes its profit over the input decisions. I.e. it solves

\[
\max_{I_t,K_t} \left\{ q_t \Psi \left( \frac{I_t}{K_t} \right) K_t - r^K_t K_t - I_t \right\}
\]
where \( r^K_t \) denotes the real rental rate on capital employed in the capital good production. The first order conditions to this problem are

\[
q_t = \frac{1}{\Psi' \left( \frac{I_t}{K_t} \right)} \tag{24}
\]

and

\[
r^K_t = q_t \left( \Psi \left( \frac{I_t}{K_t} \right) - \Psi' \left( \frac{I_t}{K_t} \right) \frac{I_t}{K_t} \right) = q_t \left( \frac{I_t}{K_t} - \frac{I_t}{K_t} \cdot \frac{I_t}{K_t} \right), \tag{25}
\]

where the last equality follows from (24).

To conclude, note first that (25) already ensures zero profits. Note further that (24) shows how changes in \( q \) are driven by adjustment costs and gives rise to its interpretation as Tobin (1969)'s \( q \).\(^{19}\)

2.3 Public sector

In some period \( t \), the government collects real taxes \( T_t \), consumes of real quantity \( e^{G_t} \) and issues bonds of nominal volume \( B_{t+1} \) which pay the predetermined nominal interest \( Q_{t+1} \). It thereby has to restrict its activity to policies that satisfy its budget constraint

\[
T_t + \frac{B_{t+1}}{P_t} \geq e^{G_t} + \frac{B_t}{P_t} Q_t. \tag{26}
\]

Furthermore, the government is not allowed to apply a Ponzi scheme to intertemporally finance its expenditures such that the debt growth rate \( \frac{B_{t+1}}{B_t} \) is capped.

2.3.1 Fiscal authority

As in Heer and Maußner (2014), government spending is exogenous. In particular, for all \( t \),

\[
G_{t+1} = (1 - \rho_G)G_{ss} + \rho_G G_t + \sigma_G \epsilon_{t+1}^G, \quad \rho_G \in [0, 1), \quad \sigma_G \geq 0, \tag{27}
\]

\(^{19}\)This is because of its resemblance to the neoclassical q-theory framework with \( q \) being the shadow price of capital, cf. e.g. Heiberger and Ruf (2014b).
where \( G_{ss} \) denotes its steady state level and \( \epsilon_t^G \sim \text{iidN}(0,1) \).

Taxes are lump sum and understood as a residual policy instrument to balance the public budget.

### 2.3.2 Monetary authority

The monetary authority on the other side affects the economy through the determination of the government bond’s nominal interest rate according to a Taylor (1993)-type policy rule. Thereby, it reacts to inflation and output, both measured as current deviations to their respective steady state values, the previous nominal interest rate and also to the current gap of Tobin’s \( q \). The allowance for such a \( q \) target in the interest rate rule justifies the expression “augmented” Taylor rule and the assessment of its potential benefit is the major concern of the present paper. Formally,

\[
Q_{t+1} = Q_t^{\delta_1} \left( \frac{\pi_{ss}}{\beta} \right)^{1-\delta_1} \left( \frac{\pi_t}{\pi_{ss}} \right)^{\delta_2} \left( \frac{q_t}{q_{ss}} \right)^{\delta_3} \left( \frac{Y_t}{Y_{ss}} \right)^{\delta_4},
\]

where

\[
(\delta_1, \delta_2, \delta_3, \delta_4) \in \{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 \in [0,1], x_2 > 1 \}.
\]

Since such a rule “[... and] include[s] as arguments only a few readily observable macroeconomic indicators and [I further demand it to] respect the zero bound on nominal interest rates [...],” it is indeed operational in the sense of Schmitt-Grohe and Uribe (2004a).²¹

### 2.4 General equilibrium

In a dynamic general equilibrium, all markets in the economy have to be cleared simultaneously with all agents acting mutually optimal at all time. In the following, I list all conditions that have to be fulfilled for such a general equilibrium.

²⁰The second parametric restriction is the here adopted implementation of the requirement for operational monetary policies to give rise to a unique equilibrium solution. It specifically demands the interest rate regime to obey the Taylor principle in that it demands the inflation target coefficient to be larger than unity such that the policy exhibits a nonzero effect on the real interest rate.

First, to avoid confusion, I introduce a separate symbol for the household’s maximum lifetime utility, \( V_t \). Note that it arises for utility maximizing composite good processes such that it is a function of the model’s state variables and satisfies the generalized Bellman recursion for all \( t \), i.e.

\[
V_t = \left[ (1 - \beta)u_t^{1 - \frac{1}{\psi}} + \beta_t \left( \mathbb{E}_t [V_{t+1}^{1 - \gamma}] \right)^{1 - \frac{1}{\psi}} \right]^{\frac{1}{1 - \psi}}.
\]

Therefore, in order to state the equilibrium conditions, I redefine the stochastic discount factor for all \( \tau \geq 0 \) to

\[
m_{t+\tau,t} := \prod_{j=0}^{\tau} \left( \beta_t \left( \frac{V_{t+j}}{\mathbb{E}_{t+j-1} [V_{t+j}^{1 - \gamma}]} \right)^{\frac{1}{\psi} - \gamma} \right) \left( \frac{u_{t+\tau}}{u_t} \right)^{1 - \frac{1}{\psi}} \frac{C_t - C_h}{C_{t+\tau} - C_{t+\tau}^h}.
\]

and, accordingly,

\[
m_{t+\tau,t}^\eta := \eta^\tau m_{t+\tau,t}.
\]

Now, additionally, for a general equilibrium the following list of conditions must be satisfied for all \( t \): (1), (2), (3), (4), (5), (6), (7), (8), (9), (14), (15), (17), (18), (19), (20), (21), (22), (23), (24), (25), (27) and (28)\(^{22}\). Next, I parallel Carlstrom and Fuerst (1997) and assume for all \( t \)

\[
\Delta_t = \alpha_\Delta \Omega_t \tilde{Y}_t, \ \alpha_\Delta \in [0, 1].
\]

Next, I follow Heer and Maußner (2014) and assume

\[
B_t = 0, \forall t,
\]

such that the government’s budget constraint (26) reduces to

\[
T_t \geq e^{G_t},
\]

which I moreover require to be satisfied with equality in all \( t \).

Further, note that both the firms in primary production and the household receive capital rents from firms in primary production and the capital goods agency. Accordingly, I use the definitions

\[
r_t = r_t^P + r_t^K
\]

\(^{22}\)Note that I do not explicitly refer to the definitional equations for \( u_t, C_h, MRS_t, NW_t \) and \( \tilde{Y}_t \) and the auxiliary equation (12) again.
to eliminate for $r_t$ and furthermore

$$D_t = D_t^P + D_t^I$$

to eliminate $D_t$ in the household’s budget constraint. Also, note that

$$K_t = K_t^H + X_t, \forall t,$$

and the economy’s capital accumulation reads

$$K_{t+1} = (1 - \delta)K_t + \Psi\left(\frac{I_t}{K_t}\right)K_t. \quad (31)$$

Moreover, the (inter)sectoral equilibrium conditions have to hold. In particular, for all $t$

$$N_t = N_t^d, \quad (32)$$
$$N_t^s = s^n_t N_t, \quad (33)$$
$$Y_t^s = s^n_t Y_t^d, \quad (34)$$
$$Y_t^P = Y_t^s. \quad (35)$$

With these conditions, the household’s budget constraint and the public budget constraint together imply (cf. appendix) the economy’s constraint

$$Y_t^d = C_t + I_t + e^{G_t} + g_t \bar{Y}_t \Phi(\bar{\omega}_t), \forall t, \quad (36)$$

where the very last summand makes explicit the waste of resources due to moral hazard in the credit relationship in primary production.

### 3 Numerical analysis

#### 3.1 Solution method

I solve for the approximation of the policy functions and the dynamic by perturbation à la Schmitt-Grohe and Uribe (2004b). Thus, I need to define

\footnote{How to perturb Epstein and Zin economies is demonstrated at length in Heiberger and Ruf (2014a). Note that Caldara et al. (2012) show that perturbation is an appropriate approximation technique in (basic) Epstein and Zin environments.}
an auxiliary control variable for the expected next period maximum lifetime utility

\[ W_t := E_t \left[V_{t+1}^{1-\gamma}\right]. \]

Moreover, I need to at least find a second order perturbation in order to be able to differentiate between different policy regimes by means of welfare comparisons\(^\text{24}\).

The actual solution of the second order Taylor approximations’ coefficients is computed using the generalized Schur (QZ) decomposition as provided by the Maple-Matlab toolbox introduced in Heiberger and Ruf (2014a). For the purpose of the present paper, it was extended and now importantly features the diagonal balancing technique of Lemonnier and van Dooren (2006) to avoid numerical issues as described in Heiberger, Klarl, and Maßner (2014)\(^\text{25}\).

3.2 Calibration

The model is calibrated at a quarterly frequency. As it is usual practice, I decompose the set of model parameters into fixed and free. The former are mostly chosen as in Heer and Maßner (2014) and Schmitt-Grohe and Uribe (2004a) and are thus, for the most part, empirically backed by the estimation results reported in Christiano, Eichenbaum, and Evans (2005). The calibration of the preference shocks’ persistence follows recent work by Basu and Bundick (2012), who draw from estimation results of Ireland (2003, 2011)\(^\text{26}\).

The value for \(\psi\) is taken from Gourio (2012). Note that the parameterization of the elasticity of intertemporal substitution (EIS) is still a matter of controversy\(^\text{27}\). However, I rely on the microfoundation of discounting shocks

\(^{24}\)See the description of the numerical treatment later. Also see Heiberger and Ruf (2014a) and Caldara et al. (2012) for some more general remarks on this issue.

\(^{25}\)See the appendix for more detailed remarks on the specific changes to the Heiberger and Ruf (2014a) toolbox.

\(^{26}\)Note that these authors only consider the discount shock. I additionally demand the MRS shock to have identical persistence. After all, the numerical results were not particularly sensitive with respect to these two parameters.

\(^{27}\)The empirical evidence in Hall (1988) suggests a value close to zero. This is e.g. supported by the analysis in Heiberger and Ruf (2014b), who show that a large class of RBC models can very well replicate empirical evidence of asset markets and the business cycle when equipped with an Epstein and Zin (1989) utility representation and a very low EIS. On the other hand, e.g. Bansal and Yaron (2004) focus on the case of \(\psi = 1.5\) and strongly argue in favor of \(\text{EIS} > 1\) within their framework. Third, e.g. Caldara et al. (2012), avoid
as provided by the Gourio (2012) framework which generates a decrease in investment as a reaction to higher disaster risk only if $\psi > 1$.

Further, the calibration of $\gamma$ partly also follows this author’s reasoning. Referring to Swanson (2012), Gourio (2012), p. 5, notes that “risk aversion is $[\nu \gamma]$.” In particular, I provide results for both alternative calibrations, $\gamma \in \{2, 12\}$. While the former is chosen conservatively to closely match the analysis in Heer and Maußner (2014), the $\gamma = 12$ case follows Gourio’s remark again parameterizing his suggested notion of risk aversion to lie around 2. This has mainly two reasons. First, the literature still does not seem to have reached a consensus on the question of with respect to what variable risk aversion should actually be calibrated in a model like the present one. Second, this gives me the opportunity to also discuss the model’s sensitivity with respect to $\gamma$. Note that I calibrate a preference for earlier resolution of composite good uncertainty in both cases $\gamma \in \{2, 12\}$.

Finally, the preference parameters $\eta$ and $\nu$ are determined in the deterministic steady state as demonstrated below. The fixed parameter’s calibration is summarized in table 1.

The remaining parameters $\sigma_\theta$ and $\sigma_\vartheta$ are considered free within certain boundaries and chosen in order to optimize the model’s empirical performance with respect to the six empirical U.S. targets used by Heer and Maußner (2014). These targets are the equity premium ($EP$) in percentage notation, the relative volatility of investment to output ($s_i/s_y$), working hours to output ($s_n/s_y$) and wages to output ($s_w/s_y$) as well as the correlation between output and working hours ($r_{yn}$) and wages and working hours ($r_{yw}$) at the numerical target values displayed in table 2. The basic point of orientation for these two parameters’ intervals is again the baseline calibration reported in Basu and Bundick (2012). I accordingly allow the preference shocks’ standard deviations to range from 0 to 0.035.

The optimization itself is carried out discretely via an equidistant grid at the interest rate rule $\delta_1 = \delta_3 = \delta_4 = 0, \delta_2 = 1.5^{31}$ taking a clear stand on this issue and report results for values of EIS smaller and larger than unity.

Note the remarks in Rudebusch and Swanson (2012) on the magnitude of the risk aversion parameter.

The steady state value of $\nu$ is about 0.175.

All variables are understood as HP-filtered real macroeconomic aggregates.

This grid approach is analogous to the welfare optimization across monetary policy regimes described below. This particular Taylor rule is used for calibration purposes because of its empirical plausibility. See Schmitt-Grohe and Uribe (2004A) and the references.
Table 1: Fixed parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
<td>Capital share</td>
</tr>
<tr>
<td>$\alpha_{\Delta}$</td>
<td>0</td>
<td>Subsidy to firms</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.03^{-0.25}</td>
<td>Impatience</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>{2, 12}</td>
<td>Risk aversion</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Depreciation</td>
</tr>
<tr>
<td>$\epsilon_n$</td>
<td>21</td>
<td>Wage-elasticity of demand for specific labor type</td>
</tr>
<tr>
<td>$\epsilon_y$</td>
<td>6</td>
<td>Price-elasticity of demand for specific intermediate good type</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>2.5</td>
<td>Capital adjustment costs</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.25</td>
<td>Monitoring costs</td>
</tr>
<tr>
<td>$\pi_{ss}$</td>
<td>1.042^{0.25}</td>
<td>Steady state inflation rate</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>0.9</td>
<td>Persistence of discount shock</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>0.9</td>
<td>Persistence of MRS shock</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.87</td>
<td>Persistence of government expenditure shock</td>
</tr>
<tr>
<td>$\rho_Z$</td>
<td>0.856</td>
<td>Persistence of common TFP shock</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.016</td>
<td>Std. deviation of government expenditure shock</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
<td>0.0064</td>
<td>Std. deviation of common TFP expenditure shock</td>
</tr>
<tr>
<td>$\varphi_n$</td>
<td>0.64</td>
<td>Fraction of labor types not allowed to optimize</td>
</tr>
<tr>
<td>$\varphi_y$</td>
<td>0.6</td>
<td>Fraction of intermediary goods not allowed to optimize</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.65</td>
<td>Consumption habit</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2</td>
<td>EIS</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>1</td>
<td>Unconditional expectation of idiosyncratic TFP component</td>
</tr>
</tbody>
</table>

3.3 Steady state

In this section, I summarize the relevant assumptions under which the deterministic steady state is solved. Its actual solution is delegated to the appendix.

**Working hours** I follow [Prescott (1986)](https://doi.org/10.2307/2286833) and set

$$N_{ss} = 0.33.$$ cited therein.
Table 2: Empirical targets

<table>
<thead>
<tr>
<th>EP</th>
<th>$s_i/s_y$</th>
<th>$s_n/s_y$</th>
<th>$s_w/s_y$</th>
<th>$r_{yn}$</th>
<th>$r_{wn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.18</td>
<td>2.97</td>
<td>0.98</td>
<td>0.44</td>
<td>0.78</td>
<td>0.21</td>
</tr>
</tbody>
</table>

**Capital accumulation**  With respect to the adjustment cost function $\Psi$, I use the [Jermann (1998)](#) specification

$$\Psi(x) = \frac{\delta}{1-\zeta} x^{1-\zeta} - \frac{\lambda}{1-\zeta} , \quad \zeta > 1$$

for the numerical computations. Moreover, I set

$$I_{ss} = \delta K_{ss} \quad \text{(37)}$$

in order to make the standard model’s investment steady state emerge. Hence,

$$q_{ss} = 1.$$  

**Credit contract**  The optimal contract as derived in subsection 2.2.1 determines the return on credit lending $r_t^L$. Since, intratemporally, internal financing has an interest of unity, this is equivalent to the external finance premium $efp$, i.e.

$$efp_t = \frac{r_t^L}{1}.$$  

Hence, by the definition of $\bar{\omega}_t$

$$efp_t = \bar{\omega}_t \frac{g_t e^{z_t} k_t^{\alpha} N_t}{M_t - NW_t} \overset{\text{10}}{=} \bar{\omega}_t \frac{g_t e^{z_t} k_t^{\alpha} N_t}{M_t v_t g(\bar{\omega}_t)} \overset{\text{12}}{=} \bar{\omega}_t \frac{\tilde{\omega}_t}{g(\bar{\omega}_t)}.$$  

Following [Heer and Maußner (2014)](#), I demand for the deterministic steady state that

$$efp_{ss} = 1.0187^4. \quad \text{(38)}$$

Notes:

1. Note that this choice is consistent with the above characterization of $\Psi$. 

29
Idiosyncratic TFP  I assume for all $t$ and $f \in [0,1]$ that $\omega_t(f)$ is distributed iid lognormally, i.e. that $\phi$ and $\Phi$ are the lognormal pdf and cdf, respectively. In particular, I assume

$$\ln(\omega_t(f)) \sim \text{iidN}(\mu_\omega, \sigma_\omega),$$

with

$$\mathbb{E}[\omega_t(f)] = e^{\mu_\omega + \frac{\sigma_\omega^2}{2}} = \Omega_t = \Omega = 1.$$ 

I.e., equivalently,

$$\mu_\omega + \frac{\sigma_\omega^2}{2} = 0.$$ (39)

Furthermore, as in Heer and Maußner (2014), I fix a steady state default probability of 0.00974. I.e.

$$P(\omega_t(f) \leq \bar{\omega}_{ss}) = \Phi(\bar{\omega}_{ss}) = 0.00974.$$ 

I.e., with $F_{N(0,1)}(\cdot)$ as the standard normal distribution function, I require

$$F_{N(0,1)}\left(\frac{\ln(\bar{\omega}_{ss}) - \mu_\omega}{\sigma_\omega}\right) = 0.00974.$$ (40)

Government spending  I follow Heer and Maußner (2014) and assume that steady state public expenditures make up for 16% of steady state output,

$$e^{G_{ss}} = 0.16 \cdot \bar{Y}_{ss}.$$ 

3.4 Results

In this section, I first present the empirical performance of the model under the Taylor rule $\delta_1 = \delta_3 = \delta_4 = 0, \delta_2 = 1.5$. Thereafter, I report on the welfare effects of generalizing the benchmark regime, particularly allowing for a q-target in the interest rate rule.

3.4.1 Empirical performance

The empirical performance of the model is documented in comparison to the Heer and Maußner (2014) framework. I first present three benchmark simulation results. These are obtained using the original Heer and Maußner (2014) model (B1) and also using the current framework’s two closest equivalents
originating from the parameterizations $(\sigma_\theta \ \sigma_\vartheta) = (0 \ 0)$ and $\psi \in \{0.5, 2\}$. Where the former case (B2) makes the classic representation of additively separable expected utility arise and is thus best comparable to the Heer and Maußner (2014) results, the latter case (B3) serves as an actual benchmark for the upcoming calibrations as it features the actually chosen value of EIS for the case of nonzero preference shock volatility.

The simulation results stem from an average of 300 time series, each of length 80, generated by the economy’s approximate policies and dynamics that are driven by series of independent $\mathcal{N}(0, 1)$ distributed pseudorandom shocks to the model’s exogenous states $\vartheta_t, \theta_t, Z_t$, and $G_t$. The benchmark results are summarized in table 3. This table features the “empirical” probability of violating the zero lower interest bound ($empProb$) as the fraction of actual violations in the simulations next to a standard sum of squared distances “goodness of fit” measure ($Score$).

Note that the model’s moments are computed from HP-filtered simulated time series and that the return series are computed along the household’s investment Euler equations (3) and (4) as annualized time series averages such that the risk premium is actually computed ex post.

<table>
<thead>
<tr>
<th>$EP$</th>
<th>$s_i/s_y$</th>
<th>$s_n/s_y$</th>
<th>$s_w/s_y$</th>
<th>$r_{yn}$</th>
<th>$r_{wn}$</th>
<th>$Score$</th>
<th>$empProb$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>6.18</td>
<td>2.97</td>
<td>0.98</td>
<td>0.44</td>
<td>0.78</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>0.02</td>
<td>0.72</td>
<td>1.40</td>
<td>0.47</td>
<td>0.12</td>
<td>-0.02</td>
<td>43.68</td>
</tr>
<tr>
<td>B2</td>
<td>0.03</td>
<td>0.98</td>
<td>1.38</td>
<td>1.08</td>
<td>-0.05</td>
<td>-0.44</td>
<td>43.42</td>
</tr>
<tr>
<td>B3</td>
<td>0.02</td>
<td>0.70</td>
<td>1.13</td>
<td>1.07</td>
<td>-0.11</td>
<td>-0.45</td>
<td>44.76</td>
</tr>
</tbody>
</table>

Some remarks on this table follow. First, the comparison of B1 and B2 shows how the chosen Cobb-Douglas composite good aggregator makes

---

33This choice for the goodness of fit metric follows e.g. Heer and Maußner (2013) and Uhlig (2007). For some remarks, see Heiberger and Ruf (2014b).
the model perform considerably worse with respect to labor market statistics. Only the resulting equity premium is a little larger than in the Heer and Maußner (2014) framework such that the overall score does not suffer accordingly. This rise in the risk premium, however, comes along with a sharp increase in the violation probability of the zero bound. Second, this effect vanishes in the third benchmark calibration \textbf{B3} leading to the worst score. Moreover, note the acyclicity of real wages. This shortcoming, not uncommon to New Keynesian models featuring wage stickiness, is shared by all considered variants.

Next, for the calibration as described in subsection 3.2, table \textbf{4} summarizes the best data fit actually achievable for the present framework if I allow for any magnitude of the probability of a zero lower bound violation (ZLBV). The score minimizers were found at $\sigma_\vartheta = 0.0094$, $\sigma_\theta = 0.03$ for the case of $\gamma = 2$ and at $\sigma_\vartheta = 0.00325$, $\sigma_\theta = 0.006$ for the $\gamma = 12$ case.

<table>
<thead>
<tr>
<th>$EP$</th>
<th>$s_i/s_y$</th>
<th>$s_n/s_y$</th>
<th>$s_w/s_y$</th>
<th>$r_{yn}$</th>
<th>$r_{wn}$</th>
<th>Score</th>
<th>empProb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.18</td>
<td>2.97</td>
<td>0.98</td>
<td>0.44</td>
<td>0.78</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma = 2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.19</td>
<td>3.20</td>
<td>1.43</td>
<td>0.77</td>
<td>0.74</td>
<td>-0.51</td>
<td>0.88</td>
<td>0.41</td>
</tr>
<tr>
<td>$\gamma = 12$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.11</td>
<td>2.23</td>
<td>1.14</td>
<td>0.95</td>
<td>0.06</td>
<td>-0.38</td>
<td>1.69</td>
<td>0.21</td>
</tr>
</tbody>
</table>

The improvement in empirical performance caused by the two preference shocks is remarkable. Essentially, the latter allow to work on both, the equity premium mostly along $\sigma_\vartheta$ and the labor market statistics mostly along $\sigma_\theta$. In that sense, the discount shock really is an “Equity Premium Shock” as titled by Smets and Wouters (2003). Note that for the larger risk aversion case, the empirical equity premium is already replicable for much less variation in the

---

34 This is, of course, due to the chosen score criterion that puts considerably weight on the equity premium due to its percentage notation.

35 See Huang, Liu, and Phaneuf (2004) for details on both the empirical measurement and its replication in New Keynesian models.
discount shock. This has two consequences. First, the implied probability of violating the zero bound is less extrem (however still much too high). Second, the beneficiary effects of further increasing the variability of the MRS shock with respect to the labor market statistics do not balance as nicely with its negative effects as in the case of simultaneously higher values of $\sigma_\vartheta$ rendering the overall score worse.

Eventually, as summarized in table 5, the empirical potential of the model suffers dramatically if attention is restricted to calibrations that give rise to equilibria interest rates which do not violate the zero lower bound (noZLBV) in considerably more than 1% of the times under the simple benchmark Taylor rule. The respective best fits were found at $\sigma_\vartheta = 0.0011$, $\sigma_\theta = 0.01$ for the case of $\gamma = 2$ and at $\sigma_\vartheta = 0.0009$, $\sigma_\theta = 0.01$ for the $\gamma = 12$ case.

Table 5: Best achievable performance (noZLBV)

<table>
<thead>
<tr>
<th>EP</th>
<th>$s_i/s_y$</th>
<th>$s_n/s_y$</th>
<th>$s_w/s_y$</th>
<th>$r_yn$</th>
<th>$r_wn$</th>
<th>Score</th>
<th>empProb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>6.18</td>
<td>2.97</td>
<td>0.98</td>
<td>0.44</td>
<td>0.78</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>$\gamma = 2$</td>
<td>0.07</td>
<td>0.90</td>
<td>1.23</td>
<td>1.00</td>
<td>0.16</td>
<td>-0.50</td>
<td>42.87</td>
</tr>
<tr>
<td>$\gamma = 12$</td>
<td>0.39</td>
<td>0.84</td>
<td>1.23</td>
<td>1.00</td>
<td>0.16</td>
<td>-0.50</td>
<td>39.27</td>
</tr>
</tbody>
</table>

The role of risk aversion As it can be seen in the above tables, the risk aversion parameter $\gamma$ plays a significant role with respect to the empirical performance of the model. In a more specific examination it becomes evident that it has a large impact on the equity premium while it is essentially ineffective on the targeted business cycle statistics. While this is in line with the results of Tallarini (2000) and Rudebusch and Swanson (2012), it can also be seen as an extension to the analysis in Heiberger and Ruf (2014a) who suspect the risk aversion parameter to have a rather discreet effect on

---

36 I actually allowed for a violation probability of maximal 2%. However, the resulting scores did not improve by much in that region so I decided to leave some room for the upcoming welfare analysis.
the replication of empirical macroeconomic targets in a second order perturbation but expect the EIS to be the key driver. This is reconciled with the present findings by the fact that their conjecture stays also valid in the current framework for the case of $\sigma_\vartheta = 0$, i.e., in the absence of the discounting shock. In fact, the presence of discounting shocks—memorizing their microeconomic foundation of time varying disaster risk—seems to induce a considerably larger degree of sensitivity with respect to the risk aversion parameter, at least in the (relative) evaluation of assets. The impact of the risk aversion parameter $\gamma$, exemplified at a moderate discounting shock volatility of 0.0025, is displayed in figure 1. It shows, akin to the finding of Rudebusch and Swanson (2012), that the equity premium (EP, solid red line) grows roughly linearly with $\gamma$ such that any magnitude of the EP would be achievable through changing $\gamma$ accordingly. The figure compares this effect to the impact of $\sigma_\vartheta$ within the interval of actually considered values at $\gamma = 2$. This effect is of a rather exponential nature. Additionally, both graphs feature the implied equilibrium empirical zero bound violation probabilities (e.Pr., green dashed line) and show considerably higher sensitivity with respect to $\sigma_\vartheta$ in the same EP region.

Figure 1: Key parameters for the equity premium

As a consequence, the prominent role of the risk aversion parameter in the earlier literature attempting to generating sizeable equity premia would basically stem from its reciprocal linkage to the EIS in the paradigmatic framework of additively separable expected utility.

Note that in the figure the discounting shock volatility $\sigma_\vartheta$ is denoted by “sigma[disc]” and $\gamma$ by “gamma”.

Note that the effect is not monotonically. Especially in the left subfigure the graph of the empirical violation probability displays a local maximum.
In summary, since the equity premium is driven by the covariance between the return on equity and the stochastic discount factor, the results suggest that the model is not capable of generating a strong enough countercyclicality between these two variables unless I force excess variability into their relationship via the discounting shock volatility. However, the zero lower bound does not allow for the necessary magnitude of \( \sigma_0 \) and thus renders the model clearly at odds with empirical asset pricing evidence.

Eventually, as in Heer and Maussner (2014), the model—in accordance with U.S. empirical evidence—generates an anticyclical external finance premium without additionally assuming a spillover from common to idiosyncratic productivity as in Faia and Monacelli (2007).

### 3.4.2 Q-targeting

In order to assess the impact of allowing the monetary authority to employ an augmented Taylor rule such that it can also explicitly react to asset price movements, I parallel the approach of Heer and Maussner (2014). Particularly, in order to compare the welfare effects of different policy rules I follow the Lucas (1987) tradition and discriminate by means of the fraction \( \lambda_C^a \) of consumption—associated with some benchmark policy regime—that the representative household would need to forego permanently in order to be indifferent between the latter and the respective Taylor rule scenario \( a \) under consideration. More precisely, denoting the benchmark regime by a bar, \( \lambda_C^a \) is implicitly defined by

\[
V_t^a = (1 - \lambda_C^a)\nu_t \cdot \bar{V}_t
\]

Accordingly, within a set of considered interest rate rules \( A \subset \mathbb{R}^4 \), a regime \( a \in A \) is considered optimal if it yields the largest welfare enhancement relative to the considered benchmark scenario. To put it more precisely, \( a \in A \) is an optimal monetary policy regime if it holds for any \( b \in A \) that \( \lambda_C^a \leq \lambda_C^b \).

Now, to find the optimal augmented Taylor rule, I proceed as Heer and Maussner (2014) and define the benchmark regime to be the simple Taylor rule.

---


41 Find this result derived in the appendix.
rule with Q_{t+1} = \left( \frac{\pi_t}{\pi_{ss}} \right)^{1.5}.

With respect to this simple Taylor rule, I find the optimal q-targeting regime in the parametric class

\[ Q_{t+1} = Q^{\delta_1} \left( \frac{\pi_{ss}}{\beta} \right)^{1-\delta_1} \left( \frac{\pi_t}{\pi_{ss}} \right)^{\delta_2} \left( \frac{q_t}{q_{ss}} \right)^{\delta_3} \left( \frac{Y_t}{Y_{ss}} \right)^{\delta_4}. \] (41)

Note that this equation follows from the fact that \( q_{ss} = 1 \), which is exactly the value of \( q_t \) in absence of the capital adjustment friction, i.e. its efficient value. In that sense, the central bank’s policy can also be understood as targeting the asset price gap.

The approach is a discrete optimization over an equidistant grid \( A \) for the four policy parameters \( \delta_1, \delta_2, \delta_3, \delta_4 \) that determine the interest rate regime. In particular, I discretely minimize \( \lambda_C^a \), approximated at the risky steady state, over all calibrations \( a \in A \). Thereby, the basic grid intervals are chosen as in Heer and Maußner (2014)

\[ \delta_1 \in [0, 0.95], \]
\[ \delta_2 \in [1.2, 2.5], \]
\[ \delta_3 \in [-2.5, 2.5], \]
\[ \delta_4 \in [0, 2.5]. \]

I consider 5 different values for the inertia parameter \( \delta_1 \) and the inflation targeting coefficient \( \delta_2 \), 20 different values for \( \delta_3 \), the key parameter under consideration, and another 10 values for the output coefficient \( \delta_4 \). This generates a grid size of 5,000 monetary policy regimes for each risk aversion scenario \( \gamma = \{2, 12\} \).

As mentioned above, I thereby disregard any regime \( a \in A \) that violates the (theoretical) zero lower interest boundary with an undesirably high probability. Table 6 summarizes my results. Note that the minimizing \( \lambda_C^{min} \) is already displayed in percentage notation.

---

42 Again, this simple rule as a “factual” benchmark can be motivated by its empirical support in U.S. data.

43 See Faia and Monacelli (2007) for more detailed remarks.

44 An important advantage of such a grid approach is that it does not get stuck in local optima in the welfare surface. Cf. Schmitt-Grohe and Uribe (2004a).

45 See the appendix for the computation of \( \lambda_C^a \) and further details on the implemented zero lower bound threshold.

---

36
Table 6: Optimal q-targeting regime

<table>
<thead>
<tr>
<th>γ</th>
<th>δ1</th>
<th>δ2</th>
<th>δ3</th>
<th>δ4</th>
<th>λq\text{min}</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>1.85</td>
<td>-0.13</td>
<td>0.28</td>
<td>-0.009</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1.2</td>
<td>-0.13</td>
<td>0</td>
<td>-0.008</td>
</tr>
</tbody>
</table>

First, both risk aversion scenarios draw a very similar picture supporting the robustness of the overall results. In particular, in accordance with the findings of Faia and Monacelli (2007) and Heer and Maußner (2014), the monetary authority optimally reacts with a negative coefficient to asset price movements. I.e., in the case of rising asset prices, the central bank is advised to decrease the economy’s interest rate in order to improve on welfare. In fact, over all considered regimes with an explicit q-target, only those with \( δ_3 < 0 \) turned out to be welfare enhancing. Now, Faia and Monacelli (2007) interpret this result as the central bank limiting the distortional effect of a rise in \( q \) which renders the level of investment lower than in a scenario without endogenous movement in \( q \). Thereby, note that in their model such asset price movements arise from the credit friction whereas in the present framework, changes in \( q \) are due to adjustment costs in capital production. Accordingly, reacting to asset price movements is primarily reacting to the adjustment cost friction.

On the other hand, the equilibrium conditions might suggest yet another mechanism at work. A rise in \( q \) should reduce the household’s return on equity such that, by the Euler-Lucas equation for saving in the capital stock (3), his stochastic discount factor must rise.\(^{46}\) As a consequence, by (3), the real return on government bonds has to fall as well such that the nominal interest rate has to fall or inflation has to rise. Therefore, by lowering \( Q \), the central bank reduces the pressure on prices to rise. That way, a negative q target coefficient may complement ex post inflation targeting through \( δ_2 > 1 \) by an anticipatory component.

Moreover, while the considered grid contained welfare improving regimes with active interest rate smoothing and output targeting, in the optimal

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\(^{46}\)By standard intuition, this means that the household shifts (composite good) consumption to the current period. Note, however, that the assumed preference for early uncertainty resolution complicates such reasoning.
regime both are (virtually) absent. In particular, $\delta_4 = 0$ is in line with the results in Faia and Monacelli (2007) and Schmitt-Grohe and Uribe (2007).

Second, as in Faia and Monacelli (2007) and Heer and Maußner (2014), the achievable welfare effects are very small such that, from a welfare point of view, strict (ex post) inflation targeting turns out to already be too strong a benchmark. There is thus no prominent role for $q$-targeting despite the financial friction and despite of the additional shocks studied in the present work. In fact, note that through its positive effect on net worth in the primary production sector, a rise in $q$ also moderates the financial friction by lowering the bankruptcy threshold. On the one hand, the poor welfare gain is suggestive of the relative irrelevance of additional monetary targets besides inflation. On the other hand, this conclusion may be challenged in a framework that is able to generate severe asset price misalignments as witnessed before the collapse of the financial markets in 2008.

Eventually, I would like to note that besides the fact that the model showed a rather high sensitivity in some parameter regions with respect to the quality of the solution, which would in general suggests to check for robustness with respect to the approximation technique, I do not expect a different assessment regarding the importance of an explicit $q$-target. Moreover importantly, the bottom line results do not hinge on the allowed magnitude of the zero bound violation probability.

4 Conclusion

I analyzed a medium scale New Keynesian macroeconomy featuring a Carlstrom and Fuerst (1997) financial friction next to capital adjustment costs, preference shocks to the marginal rate of substitution and the impatience parameter as well as a general recursive preference representation as in Epstein and Zin (1989) with respect to both its empirical performance and its implications towards optimal monetary policy.

47 Note that the chosen approach even biases the assessed relevance of $q$-targeting upwards in that its welfare consequences are compared to those of a simple rule in spite of a more general regime within the class with only $\delta_3 = 0$ as an additional restriction.

48 Note that Heer and Maußner (2014) demonstrate the welfare relevance of the financial friction in the present framework.

49 After all, the model is far more complicated and parameter intensive as the benchmark economy Caldara et al. (2012) used in order to demonstrate the appropriateness of the perturbation approach in Epstein and Zin environments.
With respect to empirical performance, the considered preference shocks are found to be able to dramatically improve the model’s data fit. Yet, their effect virtually vanishes as soon as a plausible zero bound on the government’s nominal debt interest is imposed. Besides, the risk aversion parameter’s impact on the equity premium is (re)activated within the chosen Epstein and Zin specification by the presence of the discounting shock.

With respect to q-targeting, the results advise the central bank to respond negatively to asset price movements. The associated welfare enhancement is however hardly significant against a simple anti-inflationary benchmark. Since this result confirms the findings of Faia and Monacelli (2007) and Heer and Maußner (2014), the relevance of q-targeting is not likely to hinge on further frictions or shocks to be considered but should rather be investigated by explicitly modeling a financial crisis or an asset price bubble mechanisms that may give rise to large welfare detrimental effects to be targeted by the monetary authority.
A Appendix

This appendix provides supplementary material that is skipped in the body of the paper in order to ease exposition.

A.1 Description of the model

A.1.1 Wage setting

Necessary optimality condition First, as in Palomino and Li (2014), I define the aggregate real labor income in the household’s budget constraint separately. I.e. for some $t, \tau \in \mathbb{N}$, I write

$$l_{t+\tau} := \int_{0}^{1} \frac{W_{t+\tau}(k)}{P_{t+\tau}} N^s_{t+\tau}(k) dk = \frac{N_{t+\tau}^d W_{t+\tau}}{P_{t+\tau}} \int_{0}^{1} \left( \frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{1-\epsilon_n} dk.$$ 

Further, from the above we know that for all $k \in [0, 1]$ $N^s_{t+\tau}(k)$ depends on $\tilde{w}_t$ only if no reoptimizing has taken place after $t$. Precisely, with probability $(1 - \varphi_n)\varphi^\tau_n$,

$$N^s_{t+\tau}(k) = N_{t+\tau}^d \left( \frac{\tilde{w}_t}{w_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}.$$ 

Hence, for the representative household it follows by a law of large numbers argument that such reoptimization has not taken place from $t+1$ until $t+\tau$ for a fraction of $(1 - \varphi_n)\varphi^\tau_n \times 100\%$ of his labor types. Therefore,

$$\frac{\partial N^s_{t+\tau}}{\partial \tilde{w}_t} = (1 - \varphi_n)\varphi^\tau_n (1 - \epsilon_n) N_{t+\tau}^d \left( \frac{\tilde{w}_t}{w_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}.$$ 

Moreover, by the same reasoning,

$$\frac{\partial l_{t+\tau}}{\partial \tilde{w}_t} = (1 - \varphi)\varphi^\tau_n (1 - \epsilon_n) N_{t+\tau}^d \left( \frac{\tilde{w}_t}{w_{t+\tau} \pi_{t+\tau}} \right)^{1-\epsilon_n} \left( \frac{\tilde{w}_t}{w_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}.$$ 

Next, note that, by construction, for all $\tau$

$$\frac{\partial C_{t+\tau}}{\partial \tilde{w}_t} = \frac{\partial l_{t+\tau}}{\partial \tilde{w}_t}.$$ 

\footnote{Note that $N_{t+\tau}^s = \int_{0}^{1} N^s_{t+\tau}(k) dk$.}
such that
\[
\frac{\partial U_t}{\partial \tilde{w}_t} = U_t^\frac{1}{\gamma} \left\{ (1 - \beta) u_t^{\frac{1}{\gamma}} \left( \frac{\partial u_t \partial \lambda_t}{\partial C_t \partial \tilde{w}_t} + \frac{\partial u_t \partial N^s_t}{\partial N^s_t \partial \tilde{w}_t} \right) + \beta_t \mathbb{E}_t \left[ \left( U_{t+1}^{1-\gamma} \right)^{\frac{\gamma}{\gamma - 1}} \frac{\partial U_{t+1}}{\partial \tilde{w}_t} \right] \right\}
\]

and in any period \( t + \tau \)
\[
\frac{\partial U_{t+\tau}}{\partial \tilde{w}_t} = U_{t+\tau}^\frac{1}{\gamma} \left\{ (1 - \beta) u_{t+\tau}^{\frac{1}{\gamma}} \left( \frac{\partial u_{t+\tau} \partial \lambda_{t+\tau}}{\partial C_{t+\tau} \partial \tilde{w}_t} + \frac{\partial u_{t+\tau} \partial N^s_{t+\tau}}{\partial N^s_{t+\tau} \partial \tilde{w}_t} \right) + \beta_{t+\tau} \mathbb{E}_{t+\tau} \left[ \left( U_{t+\tau+1}^{1-\gamma} \right)^{\frac{\gamma}{\gamma - 1}} \frac{\partial U_{t+\tau+1}}{\partial \tilde{w}_t} \right] \right\}.
\]

Accordingly,
\[
\frac{\partial U_t}{\partial \tilde{w}_t} = U_t^\frac{1}{\gamma} (1 - \beta) \left\{ u_t^{\frac{1}{\gamma}} \frac{\partial u_t \partial \lambda_t}{\partial C_t \partial \tilde{w}_t} + u_t^{\frac{1}{\gamma}} \frac{\partial u_t \partial N^s_t}{\partial N^s_t \partial \tilde{w}_t} + \mathbb{E}_t \left[ \sum_{\tau=1}^{\infty} \left( \prod_{j=1}^{\tau} \beta_{t+j-1} \left( \frac{U_{t+j}}{U_{t+\tau}} \right)^{1-\gamma} \right) \left( u_{t+\tau}^{\frac{1}{\gamma}} \frac{\partial u_{t+\tau} \partial \lambda_{t+\tau}}{\partial C_{t+\tau} \partial \tilde{w}_t} + u_{t+\tau}^{\frac{1}{\gamma}} \frac{\partial u_{t+\tau} \partial N^s_{t+\tau}}{\partial N^s_{t+\tau} \partial \tilde{w}_t} \right) \right] \right\}.
\]

Thus,
\[
\frac{\partial U_t}{\partial \tilde{w}_t} = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau} \frac{\partial \lambda_{t+\tau}}{\partial \tilde{w}_t} \right] = \mathbb{E}_t \left[ \sum_{\tau=1}^{\infty} m_{t+\tau} \frac{\partial N^s_{t+\tau}}{\partial \tilde{w}_t} \right] = 0,
\]

where the equivalence follows from the strict positivity of (marginal) utility\(^{51}\).

Eventually, plugging in for the derivatives of \( \lambda_{t+\tau} \) and \( N^s_{t+\tau} \) computed above yields
\[
\tilde{w}_t = \frac{\epsilon_n \Delta_{1,t}}{\epsilon_n - 1 \Delta_{2,t}},
\]

with
\[
\Delta_{1,t} := \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau} \frac{1}{\pi_{t+\tau} w_{t+\tau}} \right] \left( \frac{\pi_t}{\pi_{t+\tau} w_{t+\tau}} \right)^{-\epsilon_n} \text{MRS}_{t+\tau},
\]

\(^{51}\)Specifically, cancel \( U_t^{\frac{1}{\gamma}} (1 - \beta) \) from the equality \( \frac{\partial u_t}{\partial \tilde{w}_t} = 0 \) and further divide it by \( u_t^{\frac{1}{\gamma}} \).
\[ \Delta_{2,t} := \mathbb{E}_t \left[ \sum_{\tau=0}^\infty \sum_{\tau=0}^\infty m_{t+\tau} N_{t+\tau}^d \left( \frac{\pi_t}{\pi_{t+\tau}} \right)^{-\epsilon_n} \left( \frac{1}{w_{t+\tau}} \right)^{-\epsilon_n} \right]. \]

The expressions for \( \Delta_1 \) and \( \Delta_2 \) can now be formulated recursively as in the body of the paper. This is finally demonstrated for \( \Delta_1 \).

\[ \Delta_{1,t} = N_t^d w_t^\epsilon \text{MRS}_t + \mathbb{E}_t \left[ \varphi_n m_{t+1,t} \sum_{\tau=0}^\infty m_{t+1+\tau} \varphi_n^\tau N_{t+1+\tau}^d \left( \frac{\pi_{t+1}}{\pi_{t+1+\tau}} \right)^{-\epsilon_n} \left( \frac{\pi_t}{\pi_{t+1}} \right)^{-\epsilon_n} \text{MRS}_{t+1+\tau} \right] \]

\[ = N_t^d w_t^\epsilon \text{MRS}_t + \mathbb{E}_t \left[ \mathbb{E}_{t+1} \left[ \varphi_n m_{t+1,t} \sum_{\tau=0}^\infty m_{t+1+\tau} \varphi_n^\tau N_{t+1+\tau}^d \left( \frac{\pi_{t+1}}{\pi_{t+1+\tau}} \right)^{-\epsilon_n} \left( \frac{\pi_t}{\pi_{t+1}} \right)^{-\epsilon_n} \text{MRS}_{t+1+\tau} \right] \right] \]

\[ = N_t^d w_t^\epsilon \text{MRS}_t + \mathbb{E}_t \left[ \varphi_n m_{t+1,t} \left( \frac{\pi_t}{\pi_{t+1}} \right)^{-\epsilon_n} \mathbb{E}_{t+1} \left[ \sum_{\tau=0}^\infty m_{t+1+\tau} \varphi_n^\tau N_{t+1+\tau}^d \left( \frac{\pi_{t+1}}{\pi_{t+1+\tau}} \right)^{-\epsilon_n} \text{MRS}_{t+1+\tau} \right] \right] \]

\[ = N_t^d w_t^\epsilon \text{MRS}_t + \mathbb{E}_t \left[ \varphi_n m_{t+1,t} \left( \frac{\pi_t}{\pi_{t+1}} \right)^{-\epsilon_n} \Delta_{1,t+1} \right]. \]

**Comparison to Heer and Maußner (2014)** In order to be able to compare the wage equation to Heer and Maußner (2014), I have to change the composite good aggregator under consideration, mute the preference shocks, and change the perspective of the optimizing instance. First, consider for all \( k \in [0,1] \) the additive aggregator

\[ u_t^a(k) := \left( \frac{(C_t - C_t^h)^{1-\frac{1}{\psi}}}{1 - \frac{1}{\psi}} - \xi_1 (N_t^\epsilon(k))^{1+\xi_2} \right)^{\frac{1}{1-\psi}}, \quad \xi_1, \xi_2 \geq 0. \]

Besides, consider the situation of a continuum of household members of unit mass, who all provide one individual labor type \( k \in [0,1] \). Moreover, let them all have an Epstein and Zin (1989) (EZ) utility representation such as the representative household in the main part of the paper but with the additional restriction \( \frac{1}{\psi} = \gamma \), which gives rise to the standard representation of additively separable expected utility.

Now, sticking with the Calvo (1983) mechanism, such a member is allowed to optimize his nominal wage in some period \( t \) with probability \( 1 - \varphi_n \).
He further provides $N^s_t(k)$ units of his labor type on which he receives individual income. Again, $N^s_{t+\tau}(k)$ depends on $\bar{w}_t$ only in one particular scenario. Precisely, with probability $(1 - \varphi_n)\tilde{v}_t^n$, it holds that

$$N^s_{t+\tau}(k) = N^d_{t+\tau} \left( \frac{\bar{w}_t(k)}{\bar{w}_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}.$$ 

The necessary optimality condition is again identical for all $k \in [0, 1]$. I thus skip $k$ and write

$$\mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t}(1 - \varphi_n)\tilde{v}_t^n (\epsilon_n - 1) N^d_{t+\tau} \left( \frac{\pi_t}{\pi_{t+\tau}} \right)^{1-\epsilon_n} \left( \frac{\bar{w}_t}{\bar{w}_{t+\tau}} \right)^{-\epsilon_n} \right] =$$

$$\mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t}(1 - \varphi_n)\tilde{v}_t^n \text{MRS}^a_{t+\tau} \epsilon_n \frac{N^d_{t+\tau} \bar{w}_t}{\bar{w}_{t+\tau} \pi_{t+\tau}} \left( \frac{\bar{w}_t}{\bar{w}_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n} \right],$$

where now the stochastic discount factor simplifies to

$$m_{t+\tau,t} := \beta^\tau \left( \frac{C_{t+\tau} - C_{t+\tau}^h}{C_t - C_t^h} \right)^{-\psi}$$

and

$$\text{MRS}^a_{t+\tau} := \frac{\epsilon_n}{\beta^\tau} \left( \frac{N^d_{t+\tau} \left( \frac{\bar{w}_t}{\bar{w}_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}}{(C_{t+\tau} - C_{t+\tau}^h)^{-\psi}} \right)^{\epsilon_n} \left( \frac{\bar{w}_t}{\bar{w}_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n}.$$ 

Thus,

$$\bar{w}_t = \frac{\epsilon_n}{\epsilon_n - 1} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} (\beta \varphi_n)^\tau \times \left( N^d_{t+\tau} \right)^{1+\epsilon_n} \left( \frac{\bar{w}_t}{\bar{w}_{t+\tau} \pi_{t+\tau}} \right)^{-\epsilon_n(1+\epsilon_n)} \right].$$

The latter expression is equivalent to the one found in Heer and Maußner (2014). In this sense, the EZ approach as employed in the present paper generalizes these authors’ analysis.

In summary, the present paper assumes a representative household who provides all labor types. This can, of course, also be interpreted as a central instance to negotiate with firms. However, the crucial point is that this

\[52\text{Note that the MRS was above defined with respect to leisure and not labor.}\]
instance chooses wages in order to optimize on welfare in terms of aggregate consumption and leisure. On the other hand, in the set up employed in Heer and Maußner (2014), different households may choose wages in order to optimize their welfare function in terms of their respective individual utility arguments. This second approach is subject to differing marginal rates of substitution across households (labor types) which can considerably complicate the decision making on the aggregate household level.

A.1.2 Wage dispersion

First, define the auxiliary variable

\[ \hat{W}_t := \left( \int_0^1 (W_t(k))^{-\epsilon_n} \, dk \right)^{-\frac{1}{\epsilon_n}} \]

such that by the same reasoning as in the derivation of the wage index equation (7)

\[ \hat{W}_t^{-\epsilon_n} = (1 - \varphi_n)\hat{W}_t^{-\epsilon_n} + \varphi_n \left( \pi_{t-1}\hat{W}_{t-1} \right)^{-\epsilon_n}. \]

Thus, with

\[ s_n^t = \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_n} \]

it follows that

\[ s_n^t = (1 - \varphi_n) \left( \frac{\hat{W}_t}{W_t} \right)^{-\epsilon_n} + \varphi_n \left( \frac{\hat{W}_{t-1}}{W_t} \right)^{-\epsilon_n} \]

\[ = (1 - \varphi_n) \left( \frac{\hat{w}_t}{w_t} \right)^{-\epsilon_n} + \varphi_n \left( \frac{\pi_{t-1}}{w_t \pi_t} \frac{\hat{w}_{t-1}}{w_{t-1} \pi_{t-1}} \right)^{-\epsilon_n} \]

\[ = (1 - \varphi_n) \left( \frac{\hat{w}_t}{w_t} \right)^{-\epsilon_n} + \varphi_n \left( \frac{w_{t-1} \pi_{t-1}}{w_t \pi_t} \right)^{-\epsilon_n} s_{t-1}^n. \]

A.1.3 Credit contract

Plugging the constraint \( y = \frac{M_t(f)}{1-v_t g(x)} \) in the objective of the optimization problem (10), differentiating the latter with respect to \( x \) and setting this

\[ 53 \text{ See Palomino and Li (2014) for some remarks.} \]
derivative to zero yields the necessary condition for optimal \( \bar{\omega}(f) \)

\[
NW_t(f) v_t \left( \frac{1 - v_t g(\bar{\omega}(f)) f'(\bar{\omega}(f)) + f(\bar{\omega}(f)) v_t g'(\bar{\omega}(f))}{(1 - v_t g(\bar{\omega}(f)))^2} \right) = 0.
\]

Because it holds for all \( t \) that \( v_t \neq 0 \) and that \( NW_t(f) > 0 \) for all \( f \in [0, 1] \), this first order condition can be equivalently stated as

\[
(1 - v_t g(\bar{\omega}(f))) f'(\bar{\omega}(f)) + f(\bar{\omega}(f)) v_t g'(\bar{\omega}(f)) = 0.
\]

Now, inserting for

\[
f'(\bar{\omega}(f)) = \Phi(\bar{\omega}(f)) - 1,
g'(\bar{\omega}(f)) = 1 - \Phi(\bar{\omega}(f)) - \kappa \phi(\bar{\omega}(f))
\]

and

\[
f(\bar{\omega}(f)) + g(\bar{\omega}(f)) = \Omega_t - \kappa \Phi(\bar{\omega}(f))
\]
yields the sought for condition.

### A.1.4 Price setting

From the above we know that, for all \( j \in [0, 1] \), \( D^I_{t+\tau}(j) \) depends on \( \hat{p}_t(j) \) only if no reoptimizing has taken place after \( t \). Precisely, with probability \((1 - \varphi_y) \varphi_y^\tau\)

\[
D^I_{t+\tau}(j) = \left( \left( \frac{\hat{p}_t(j) \pi_t}{\pi_{t+\tau}} \right)^{1-\varphi_y} - g_{t+\tau} \left( \frac{\hat{p}_t(j) \pi_t}{\pi_{t+\tau}} \right)^{-\varphi_y} \right) Y^d_{t+\tau}.
\]

Hence, for the firm in the intermediate production sector it follows by a law of large numbers argument that such reoptimization has not taken place from \( t + 1 \) until \( t + \tau \) for a fraction of \((1 - \varphi_y) \varphi_y^\tau \times 100\% \) of its product types. Therefore

\[
\text{max}_{\hat{p}(j)} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau} D^I_{t+\tau} \right]
\]

\footnote{Note that \( D^I_{t+\tau} = \int_0^1 D^I_{t+\tau}(j) dj \).}
thus reads
\[
0 = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t} (1 - \varphi_y) \varphi_y^{\tau} Y_t^{d+\tau} \left( (1 - \epsilon_y) \left( \hat{\pi}_t(j) \frac{\pi_t}{\pi_{t+\tau}} \right)^{-\epsilon_y} \frac{\pi_t}{\pi_{t+\tau}} + \epsilon_y g_{t+\tau} \left( \hat{\pi}_t(j) \frac{\pi_t}{\pi_{t+\tau}} \right)^{-\epsilon_y - 1} \frac{\pi_t}{\pi_{t+\tau}} \right) \right],
\]
which is identical for all such \(j\). Thus, skipping the index \(j\), this can equivalently be stated as
\[
(1 - \epsilon_y) \pi_t \hat{\pi}_t \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t} (1 - \varphi_y) \varphi_y^{\tau} Y_t^{d+\tau} \left( \frac{1}{\pi_{t+\tau}} \right)^{1 - \epsilon_y} \right] =
\]
\[
- \epsilon_y \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t} (1 - \varphi_y) \varphi_y^{\tau} Y_t^{d+\tau} g_{t+\tau} \left( \frac{1}{\pi_{t+\tau}} \right)^{-\epsilon_y} \right],
\]
or more compactly
\[
\hat{\pi}_t = \frac{\epsilon_y}{\epsilon_y - 1} \frac{\Gamma_{1,t}}{\pi_t},
\]
with
\[
\Gamma_{1,t} := \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t} (1 - \varphi_y) \varphi_y^{\tau} Y_t^{d+\tau} \left( \frac{1}{\pi_{t+\tau}} \right)^{-\epsilon_y} \right]
\]
and
\[
\Gamma_{2,t} := \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} m_{t+\tau,t} (1 - \varphi_y) \varphi_y^{\tau} Y_t^{d+\tau} \left( \frac{1}{\pi_{t+\tau}} \right)^{1 - \epsilon_y} \right].
\]
The expressions for \(\Gamma_1\) and \(\Gamma_2\) can eventually be formulated recursively as in the body of the paper.

A.1.5 Price dispersion

The derivation of the dynamic equation for the price dispersion variable \(s^y\) is analogous to the derivation of the wage dispersion equation above but nevertheless briefly outlined here for convenience. First, define the auxiliary variable
\[
\hat{P}_t := \left( \int_0^1 (P_t(j))^{-\epsilon_y} \, dj \right)^{-\frac{1}{\epsilon_y}}
\]
such that
\[ \hat{P}_t^{-\epsilon_y} = (1 - \varphi_y)\hat{P}_t^{-\epsilon_y} + \varphi_y \left( \pi_{t-1} \hat{P}_{t-1} \right)^{-\epsilon_y} \]
and
\[ s_t^y = \left( \frac{\hat{P}_t}{P_t} \right)^{-\epsilon_y}. \]

It thus follows that
\[ s_t^y = (1 - \varphi_y)\hat{P}_t^{-\epsilon_y} + \varphi_y \left( \frac{\pi_{t-1}}{\pi_t} \right)^{-\epsilon_y} s_{t-1}^y. \]

### A.1.6 Economy budget constraint

With the governmental constraint plugged in, the household’s constraint can be written as

\[
C_t + e^{G_t} = w_t N_t^y + (r_t^P + r_t^K)K_t^H + D_t^P + D_t^I - q_t (K_{t+1}^H - (1 - \delta)K_t^H) \\
= (1 - \alpha) \frac{g_t}{v_t} \hat{Y}_t + (r_t^P + r_t^K)(K_t - X_t) - q_t (K_{t+1} - X_{t+1} - (1 - \delta)(K_t - X_t)) + D_t^P + D_t^I \\
= (1 - \alpha) \frac{g_t}{v_t} \hat{Y}_t + (r_t^P + r_t^K)K_t - q_t (K_{t+1} - (1 - \delta)K_t) + D_t^I + \\
+ D_t^P + q_t X_{t+1} - X_t (q_t(1 - \delta) + r_t^H + r_t^P). 
\]

Now, note that
\[
D_t^P = - \int_0^1 \Delta_t(f)df - q_t \int_0^1 X_{t+1}(f)df + \int_{\{f: \omega_t(f) > \bar{\omega}_t\}} g_t e^{Z_t} k_t^\alpha N_t(f)(\omega_t(f) - \bar{\omega}_t)df \\
= - \Delta_t - q_t X_{t+1} + g_t \hat{Y}_t \int_{\{f: \omega_t(f) > \bar{\omega}_t\}} (\omega_t(f) - \bar{\omega}_t)df \\
= - \Delta_t - q_t X_{t+1} + g_t \hat{Y}_t \left( \int_{\hat{\omega}_t}^{\infty} \omega \phi(\omega)d\omega - (1 - \Phi(\bar{\omega}_t))\bar{\omega}_t \right),
\]

where the second equality makes use of the independence between \( \omega_t(f) \) and both \( N_t(f) \) and \( Z_t \). The last equality follows from a law of large numbers.
argument. With the above introduced notation, this can be written as

\[ D_t^P = -\Delta_t - q_tX_{t+1} + g_t\tilde{Y}_t f(\tilde{\omega}_t) \]

It thus follows that

\[ C_t + e^{G_t} = (1 - \alpha)\frac{g_t}{v_t}\tilde{Y}_t + (r_t^P + r_t^K)K_t - q_t(K_{t+1} - (1 - \delta)K_t) + D_t^I + \]

\[ + g_t\tilde{Y}_t f(\tilde{\omega}_t) - (X_t(q_t(1 - \delta) + r_t^R + r_t^P) + \Delta_t) \]

\[ = (1 - \alpha)\frac{g_t}{v_t}\tilde{Y}_t + (r_t^P + r_t^K)K_t - q_t(K_{t+1} - (1 - \delta)K_t) + D_t^I + g_t\tilde{Y}_t f(\tilde{\omega}_t) - NW_t, \]

where the last equality makes use of the definition of \( NW_t \). Next, by (24) and (25) it follows that

\[ (r_t^P + r_t^K)K_t = \alpha g_t\frac{v_t}{v_t}\tilde{Y}_t + q_t\Psi \left( \frac{I_t}{K_t} \right) K_t - I_t. \]

Thus,

\[ C_t + e^{G_t} + I_t = \frac{g_t}{v_t}\tilde{Y}_t - q_t \left( K_{t+1} - (1 - \delta)K_t - \Psi \left( \frac{I_t}{K_t} \right) K_t \right) + D_t^I + g_t\tilde{Y}_t f(\tilde{\omega}_t) - NW_t, \]

where the second summand on the right hand side vanishes due to the economy’s capital accumulation.

Moreover, from (10) it follows that

\[ NW_t = M_t(1 - v_t g(\tilde{\omega}_t)) = g_t\frac{v_t}{v_t}\tilde{Y}_t (1 - v_t g(\tilde{\omega}_t)). \]

Thus,

\[ C_t + e^{G_t} + I_t = g_t\tilde{Y}_t (f(\tilde{\omega}_t) + g(\tilde{\omega}_t)) + D_t^I \]

\[ = g_t\tilde{Y}_t (\Omega_t - \kappa \Phi(\tilde{\omega}_t)) + D_t^I. \]

Eventually, note

\[ D_t^I = \frac{1}{P_t} \int_0^1 P_t(j) Y_t^*(j) dj - g_t \int_0^1 Y_t^*(j) dj \]

\[ = Y_t^d - g_t \Omega_t \tilde{Y}_t, \]

\[ \text{This equation offers yet another perspective on a law of large numbers at work. Since} \]

\[ \text{this is some firm } f \text{'s expected overall profit, it equals the aggregate (i.e. average) profit} \]

\[ \text{because there is a continuum of firms in primary production with unit mass.} \]
where the last equality follows from the intersectoral equilibrium condition
\[ Y_t^s = Y_t^P. \]

It hence follows that
\[
C_t + e^{G_t} + I_t = Y_t^d - g_t \tilde{Y}_t \kappa \Phi(\bar{\omega}_t) \\
\Leftrightarrow Y_t^d = C_t + I_t + e^{G_t} + g_t \tilde{Y}_t \kappa \Phi(\bar{\omega}_t).
\]

I.e., for the case of no monitoring costs ($\kappa = 0$) the standard budget constraint emerges.

A.2 Numerical Analysis

A.2.1 Steady state

First, from (9), (1) and (2) we find
\[
Z_{ss} = 0, \quad \theta_{ss} = 0, \quad \vartheta_{ss} = 0
\]
and
\[
\beta_{ss} = \beta.
\]

Further, from its definition, the steady state $\tau$ period discount factor is $\beta^\tau$.

Moreover, from (28) it follows that
\[
Q_{ss} = \frac{\pi_{ss}}{\beta}.
\]

Next, conditions (38), (39) and (40) together constitute a system of three equations which I solve numerically for $\bar{\omega}_{ss}$ and the distribution parameters $\mu_\omega$ and $\sigma_\omega$. This then also determines $\phi(\bar{\omega}_{ss})$, $f(\bar{\omega}_{ss})$ and $g(\bar{\omega}_{ss})$.

Next, from (25) and (37) it follows that
\[
r_{Kss} = 0,
\]
while (18) yields
\[
v_{ss} = \left(1 - \kappa \Phi(\bar{\omega}_{ss}) - \frac{f(\bar{\omega}_{ss}) \kappa \phi(\bar{\omega}_{ss})}{1 - \Phi(\bar{\omega}_{ss})}\right)^{-1}.
\]
Therefore, by (3) and (19),
\[ \eta = 1 - \frac{v_{ss} g(\bar{\omega}_{ss})}{v_{ss} f(\bar{\omega}_{ss})}. \]

Next, again from (3) it now together with (15) follows that
\[ 1 = \beta \left( \frac{g_{ss}}{v_{ss}} \alpha K_{ss}^{-1} \left( N_{ss}^s \right)^{1-\alpha} + 1 - \delta \right) \]
\[ \Leftrightarrow K_{ss} = \left( \frac{v_{ss}}{g_{ss}} \cdot \frac{1 - \beta (1 - \delta)}{\alpha \beta} \right)^{\frac{1}{\alpha - 1}} N_{ss}^s. \]

Consequently, by (15)
\[ y^Y_{ss} = \frac{g_{ss}}{v_{ss}} \alpha K_{ss}^{-1} \left( N_{ss}^s \right)^{1-\alpha}, \]

by (14),
\[ w_{ss} = \frac{g_{ss}}{v_{ss}} (1 - \alpha) K_{ss}^\alpha \left( N_{ss}^s \right)^{-\alpha}, \]

by (37),
\[ I_{ss} = \delta K_{ss} \]

and by definition
\[ \tilde{Y}_{ss} = K_{ss}^\alpha \left( N_{ss}^s \right)^{1-\alpha}. \]

Accordingly,
\[ G_{ss} = \ln(0.16 \cdot \tilde{Y}_{ss}) \]

and
\[ \Delta_{ss} = \alpha \Delta \cdot \tilde{Y}_{ss} \]

are determined as well. Now, from (22) it follows that
\[ \tilde{p}_{ss} = 1 \]

such that by (23)
\[ s^y_{ss} = 1. \]

Therefore, by (34) and (35)
\[ Y_{ss}^d = \tilde{Y}_{ss}, \]

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such that
\[ C_{ss} = \tilde{Y}_{ss} - I_{ss} - e^{G_{ss}} - g_{ss}\tilde{Y}_{ss}\kappa\Phi(\tilde{\omega}_{ss}) = \tilde{Y}_{ss}(0.84 - g_{ss}\kappa\Phi(\tilde{\omega}_{ss})) - I_{ss}. \]

Hence, by definition,
\[ C_{ss}^h = \chi C_{ss} \]
and
\[ u_{ss} = (C_{ss}(1 - \chi))^{\nu}(1 - N_{ss}^s)^{1-\nu} \]
are fixed. Accordingly,
\[ V_{ss} = u_{ss} \]
and also the auxiliary variable
\[ W_{ss} = V_{ss}^{1-\gamma} \]
are determined. Moreover, from (7) it follows that
\[ \tilde{w}_{ss} = w_{ss} \]
such that by (8)
\[ s_{ss}^n = 1. \]
Consequently, by (33),
\[ N_{ss}^d = N_{ss}^s. \]
Moreover, by (5) and (6) it follows that
\[ \nu = \frac{\epsilon_n C_{ss}(1 - \chi)}{(\epsilon_n - 1)(1 - N_{ss}^s)\tilde{w}_{as} + \epsilon_n C_{ss}(1 - \chi)}, \]
\[ \Delta_{1,ss} = \frac{N_{ss}^d w_{ss}^{\epsilon_n}}{1 - \beta \varphi_n} \]
and
\[ \Delta_{2,ss} = \frac{N_{ss}^d w_{ss}^{\epsilon_n}}{1 - \beta \varphi_n}. \]
Similarly, by (20) and (21) it follows that
\[ g_{ss} = \frac{\epsilon_y - 1}{\epsilon_y} , \]
\[ \Gamma_{1,ss} = \frac{g_{ss}Y_{ss}^d w_{ss}}{1 - \beta \phi_y} \]

and

\[ \Gamma_{2,ss} = \frac{Y_{ss}^d w_{ss} \gamma_{ss}^{-1}}{1 - \beta \phi_y} \]

respectively. Further note that

\[ w_{ss}^h = w_{ss} \]

and

\[ \pi_{ss}^h = \pi_{ss} \].

Next, from (17) and (12) it follows that

\[ NW_{ss} = \frac{g_{ss}Y_{ss}^d \tilde{Y}_{ss}(1 - v_{ss}g(\bar{\omega}_{ss}))}. \]

Further, by definition,

\[ X_{ss} = \frac{NW_{ss} - \Delta_{ss}}{1 - \delta + r_{ss}}. \]

Moreover, by their respective definitions,

\[ D_{ss}^P = g_{ss}Y_{ss}^d(\bar{\omega}_{ss}) - X_{ss} - \Delta_{ss} \]

and

\[ D_{ss}^I = Y_{ss}(1 - g_{ss}). \]

### A.2.2 Zero bound

Similarly to [Heer and Maußner (2014)](#), I exclude parameterizations that imply an excess over the empirically plausible probability of the central bank violating the zero lower interest bound. More precisely, I follow the approach of [Schmitt-Grohe and Uribe (2004a)](#) and compute a threshold for policies that render the probability of a negative nominal interest rate higher than 2%. Precisely, (assuming stationarity) I demand for all \( t \)

\[ P(Q_t - 1 < 0) \leq 0.02. \]  \hfill (42)

The implementation of this condition is carried out as follows. On the one hand, I compute a “theoretical” threshold\(^56\). In particular, I follow

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\(^56\) The threshold is theoretical in a sense that it does not vary over simulations.
Heer and Maußner (2014) and assess the zero bound violation probability along $Q_t$’s first order approximation. By the certainty equivalence property of first order perturbation,\footnote{Cf. Schmitt-Grohe and Uribe (2004b). How these authors’ reasoning carries over to EZ economies is demonstrated in Heiberger and Ruf (2014a).} it follows for all $t$ that the unconditional mean of $Q_t$ equals its deterministic steady state $Q_{ss}$. Moreover is the linear approximation of $Q_t$ normally distributed due to the assumed normality of the iid shocks. Consequently, denoting $Q_t$’s standard deviation by $\sigma_Q$,\footnote{Note the stationarity of the process governing $Q_t$.} I can derive the following upper bound for $\sigma_Q$ in order to satisfy (42),

\[
P(Q_t < -1) \leq 0.02
\]
\[
\Leftrightarrow P \left( \frac{Q_t - Q_{ss}}{\sigma_Q} < \frac{1 - Q_{ss}}{\sigma_Q} \right) \leq 0.02
\]
\[
\Leftrightarrow F_{N(0,1)} \left( \frac{1 - Q_{ss}}{\sigma_Q} \right) \leq 0.02
\]
\[
\Leftrightarrow \frac{1 - Q_{ss}}{\sigma_Q} \leq F_{N(0,1)}^{-1}(0.02) \approx -2.05.
\]

Therefore, in order to be operational the policy regime $(\delta_1, \delta_2, \delta_3, \delta_4)$ is required to give rise to an interest rate standard deviation that satisfies

\[
\sigma_Q \leq \frac{Q_{ss} - 1}{2.05} \footnote{Note that -2.05 is a little larger than the factual 2% quantile such that I actually allow slightly more than a 2% probability of a zero bound violation. This follows Heer and Maußner (2014) who moreover provide a detailed demonstration of the computation of $\sigma_Q^a$ for some regime $a \in A$.}
\]

On the other hand, I compute the actual “empirical” violation probability as the relative frequency in the simulations. Assuming the number of simulations (24,000) to be a large number, this provides me with an important complement to the theoretical magnitude derived above for the counterfactual case of relying on a first order approximation. Importantly, it additionally excludes regimes that render the solution “explosive” (i.e. instable) through its second order derivatives and it allows me to additionally assess the order of magnitude of the respective policies’ zero bound violation.
A.2.3 Welfare criterion

Derivation  Formally, first define the benchmark welfare as

\[ V_t := \left( (1 - \beta) \bar{u}_t^{1 - \frac{1}{\nu}} + \beta_t \left( \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\gamma - 1}} \right)^{1 - \frac{1}{\nu}}, \]

where

\[ \bar{u}_t = (\bar{C}_t - \bar{C}_{t-1}^h)^\nu (1 - \bar{N}_t)^{1-\nu} \]

and

\[ \bar{C}_t^h = \chi \bar{C}_{t-1}. \]

Next, note that for all \( t \) the case of permanently consuming \( (1 - \lambda C) \times 100\% \) of the benchmark level yields an associated composite good of

\[ (1 - \lambda C)^\nu \bar{u}_t. \]

Furthermore, note that the employed Kreps-Porteus certainty equivalent for some period \( t + 1 \) random variable \( X_{t+1} \)

\[ \mu_{t}^{KP} (X_{t+1}) := \left( \mathbb{E}_t \left[ X_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\gamma - 1}} \]

is homogenous. It thus follows that

\[ \mu_{t}^{KP} ((1 - \lambda C)^\nu \bar{V}_{t+1}) = (1 - \lambda C)^\nu \mu_{t}^{KP} (\bar{V}_{t+1}). \]

Hence, the associated welfare of a situation in which the representative household has to permanently forego \( (1 - \lambda C) \times 100\% \) of the benchmark consumption level, denoted by \( V_t^\lambda \), satisfies

\[ V_t^\lambda = (1 - \lambda C)^\nu \cdot \bar{V}_t. \]

Consequently, the assessment of potential benefit from augmenting the Taylor rule with respect to the benchmark case reduces to computing the respective \( \lambda_C^a \) for the considered variant \( a \) from its defining condition

\[ V_t^a \equiv (1 - \lambda_C^a)^\nu \cdot \bar{V}_t, \]

i.e.

\[ \lambda_C^a = 1 - \left( \frac{V_t^a}{V_t} \right)^{\frac{1}{\gamma}}. \]

See Epstein and Zin (1989) for the definition and Heiberger and Ruf (2014a) for some additional remarks.
**Approximation** For any variant \(a \in A\), the approximation of \(\lambda^*_\xi\) is carried out from the perspective of the representative household being at the steady state level of the state variables but aware of the uncertainty in the economy, i.e. at the risky steady state

\[(x, \xi) = (x_{ss}, 1),\]

where \(\xi\) denotes the perturbation parameter. Accordingly, the welfare measure is computed along

\[
\lambda^*_\xi \approx 1 - \left( \frac{V^a_{ss} + \frac{1}{2} V^a_{\xi}(x_{ss}, 0)}{V_{ss} + \frac{1}{2} V_{\xi}(x_{ss}, 0)} \right)^{\frac{1}{\nu}}.
\]

Note that I do not approximate \(\lambda^*_\xi\) directly as Heer and Maußner (2014) but by means of the readily available risky steady state solutions of the respective regimes’ value functions. Moreover, note that it holds for all \(a \in A\) that

\[V^a_{ss} = \bar{V}_{ss},\]

which exemplifies the necessity of computing perturbations at least up to second order for such welfare analyses.

**A.3 Documentation of computation routines**

Basically, I employed the toolbox introduced in Heiberger and Ruf (2014a) and Heiberger and Ruf (2014b). However, the present work required some additions. First, the computation of the financial friction steady state is processed in the `getCFss` procedure which uses the Maple-Matlab link. Second, mostly the size of the model (12 states plus the perturbation parameter, 39 equilibrium conditions) made it necessary to modify the core perturbation routine (`getlsg_fneu_bal`) in several respects. It is now able to handle more than 10 state variables and importantly loads the symbolically derived systems of equations (as generated in `glsys`), once they have been computed, before they are evaluated at the respective calibrations’ parameter values. The latter modification allowed me to save considerable computation time in the discrete optimizations (far more than 50%). Moreover, it features the diagonal balancing technique of Lemonnier and van Dooren (2006) to improve
on the Schur decomposition’s accuracy. The programs were run on Maple 18 and Matlab 2014a.

\footnote{The current version of the Maple/Matlab toolbox can be downloaded from http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/ruf_en.html as well as http://www.wiwi.uni-augsburg.de/vwl/maussner/lehrstuhl/heiberger_en.html}
References


